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A methodology to form families of products by applying fuzzy logic

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Abstract More and more companies are designing product families with the aim of offering a wider variety of products and at the same time reducing product cost by standardizing components and processes, making mass customization a reality. This paper proposes a comprehensive methodology to form product families taking advantage of the ability of the fuzzy logic to tackle uncertainties. In this methodology, fuzzy logic is considered as a valuable tool to improve the decision-making process due to its ability to manage information more accurately than binary logic. This methodology is presented and explained through an illustrative application to demonstrate its applicability and practicality.

Keywords Product family \cdot Product configuration \cdot Fuzzy logic \cdot Fuzzy preferences \cdot Market segmentation \cdot Mass customization

1 Introduction

In recent decades, companies have applied various strategies in an attempt to be more competitive from a number of perspectives. Among them, mass customization has played an important role in the improvement of product family design, allowing greater competitiveness with respect to product variety and cost by taking advantage of the benefits of product standardization. A promising tool in product family design has been the product modularity; it makes possible the design of a variety of products using the same

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M. Bajaras e-mail: marco.bajaras@polymtl.ca component modules; it sometimes refers to platforms. In fact, according to Moon et al. [32], a product family can be defined as a group of related products based on a product platform, which facilitates mass customization by providing a variety of products cost-effectively for different market segments.

The main objective of this paper is to propose a methodology for the design of product families by using fuzzy logic throughout its different phases and steps. In this methodology, fuzzy logic, principally fuzzy preference relations, has been used in order to improve the decision making process. Fuzzy preference relations permit the analysis of information presented in linguistic terms through the use of fuzzy numbers. Fuzzy logic exceeds binary logic due to its ability to manage vague or imprecise information by using linguistic terms such as "highly important", "moderately important", "not important" among others.

This paper differs from prior studies due to the authors have minimally and partially applied fuzzy logic in their processes to solve particular aspects. This paper presents a global methodology to form families of products by developing several fuzzy logic-aided tools to improve the whole process. These tools include a procedure to perform the market segmentation, a procedure for the identification of modules, a procedure to identify alternatives of product configurations, and a procedure for a generic product's configuration.

The rest of this paper is structured as follows. Section 2 presents a literature review related to the design of product families and to the use of fuzzy logic in it. Section 3 presents the proposed methodology to form product families by applying fuzzy logic. Section 4 shows how to apply the proposed methodology through an illustrative example. Section 5 concludes the paper.

2 Literature review

2.1 Design of product families

Product family design is a powerful strategy which makes it possible to take advantage of product similarities to reduce design and manufacturing costs. Several works have been published during the last decades. For instance, Moorthy [33] proposed a theory of market segmentation based on consumer self-selection by considering a price discrimination model to design a line of products. In the same way, Kohli and Sukumar [23] proposed some heuristics methods to design a line of products by using conjoint analysis and dynamicprogramming to identify solutions that are no worse, in terms of approximating optimal solutions.

In regard to the design of product families, many works have been published. Petiot and Grognet [36] have used multidimensional scaling to develop an approach to capture the subjectivity of human assessments and to build a perceptual space to describe the perceptual attributes of a family of products. Agard and Kusiak [2] proposed a methodology to design families of products based on customer descriptions and requirements by using data mining to analyze the functional requirements, the design of a functional structure, and the design of a technical structure. Hsiao and Liu [19] presented a methodology to design families of products by managing its variety, principally based on market planning, with Quality Function Deployment. Also, Kumar et al. [24] proposed a methodology to design product families integrating market considerations to examine the impact of increasing the product variety, and to explore the cost savings associated with the use of commonality decisions.

According to Simpson [39] there are two general approaches in product family design. The first is a top-down (proactive platform) approach, wherein the company's strategy is to develop a product family based on a product platform and its derivatives. The second is bottom-up (reactive redesign) approach, wherein a company redesigns and/or consolidates a group of products to standardize components and reduce costs.

According to Messac et al. [30], the key for a successful product family is a common product platform around which the product family is derived. In this sense, an important number of works has been published including methods to identify a platform using data mining techniques and fuzzy clustering [32], methods for the platform development applying preference aggregation, optimization [11], and cluster analysis [10]. Also, Petiot and Grognet [37] have proposed a method based on vectors fields to model the preferences of customers in a perceptual space to describe the perceptual attributes of a product family. Doré et al. [13] presented an approach to integrate the user's requirements at the beginning of the design process to find the relationship between

sensorial and functional characterization of products. Furthermore, clustering and sensitivity analysis have been used to design multiple-platform configurations in an attempt to improve product family design [12]. Cluster analysis has also been applied to the design of product platforms by analyzing products designed individually and by determining the optimal number of common values for each platform [6]. Ninan [35] presented a platform cascading method for scalebased product family design aimed at reducing the poor performance of the product family due to the consideration of a single platform by instead taking into account multiple platforms.

Commonality and modularity are two strategies successfully applied in the development of product platforms [20]. A proper balance between product platform commonality and individual product performance is very important to the success of a product family. Two sources of commonality (component and process) have been identified by Jiao and Tseng [22]. For the modelling of commonality of components, two models have been presented by Mishra [31]: (1) the multiple product/multiple common component method, and (2) the multiple product/single common component method. Dai [10] proposed a method for making an appropriate commonality decision in order to achieve a meaningful tradeoff between the technical and monetary aspects of the product family and Fellini [15] and Fellini et al. [14] presented a methodology to perform commonality optimization by choosing the components of the product that are to be shared without exceeding user-specified bounds on performance and allowing the maximization of commonality at different levels of acceptable performance. In order to cluster the attributes of the product family in a platform and its associated differentiating modules, Ye and Gershenson [46] presented a methodology to identify the appropriate commonality and variety trade-off at the product attribute level using market analysis and conceptual engineering knowledge.

Moreover, modularity has also been applied successfully in product platform development. In this context, clustering has been used to analyze the design matrix to identify modules by mapping the relationships between functional requirements and design parameters [41]. In [25], Kusiak proposed different points of view for the modular design of products, processes, and systems. A method based on the simulated annealing algorithm was proposed by Wang et al. [44] to permit the development of a modular product family. Sered and Reich [38] proposed a method for modularity standardization, focusing the engineering effort on the product platform components, and Meng et al. [29] presented a methodology to identify the component modules for product families. Da Cunha et al. [9] proposed various heuristic algorithms for the design of modular elements in a mass customization context, focusing on minimizing the manufacturing and transportation cost in the supply chain.

2.2 Fuzzy logic in product design

Fuzzy logic has been applied separately in different processes related to the design of products. According to Nagamachi [34] fuzzy logic was applied in conjunction with Kansei engineering by Sanyo to make a sophisticated color copy machine. Kansei engineering is a technology for translating human feelings into a product design [21]. Fuzzy logic has been utilized to express the hue, brightness and saturation in the construction of the fuzzy model, which has been implemented as an intelligent color copy machine. Also, many attempts have been made to simplify the use of QFD. These applications include fuzzy inference techniques to accommodate possible imprecision and vagueness [17]; fuzzy outranking to prioritize design requirements [43]; fuzzy numbers to represent the imprecise nature of judgments and to define the relationships between engineering characteristics and customer attributes [42], and fuzzy regression to identify the relational functions between, and among, engineering characteristics and customer requirements [8]. These works represent the basis to develop new fuzzy logic-aided tools aiming at the improvement of the design of product families.

In the same way, fuzzy logic has been applied in market segmentation. Chen et al. [7] used fuzzy clustering to analyze company productivity, identifying clusters in training productivity patterns by using two methods, the fuzzy C-means algorithm and the fuzzy K-NN algorithm. Clustering analysis has been combined with fuzzy recognition to support product design, with a view of forming standard structural trees of products according to the design requirements [26]. Gao et al. [18] combined similarity matrix fuzzy clustering to reengineer the product interfaces by identifying the relationships between them and attempting to reduce their redundancy.

In [40], Tong and Su found that the Taguchi's loss function and the indifference curve in the technique for order preference by similarity to ideal solution (TOPSIS) have similar features. However, the Taguchi method deals with only one-dimensional problem and TOPSIS handles multi-dimensional problems. In order to optimize the multiresponse problems in the Taguchi method and to reduce the uncertainty for determining the weight of each response, these authors applied fuzzy set theory to propose a multiple attribute decision making procedure. In the same way, Mejia-Gutierrez et al. [28] have developed a methodology to support the decision making in the design process by proposing a multi-agent approach to support the distributed knowledge elicitation process. In [27], Mejia-Gutierrez et al. have proposed a multi-agent system to aid expert to define variables and constraints in the product design process. However, these authors have concluded that the variable's shared domains should be treated with fuzzy logic to ensure the resolution of conflict when two or more experts share variables.

Fuzzy clustering approaches have been proposed in the context of product families for the identification of groups of customers with similar preferences with the purpose of designing the proper set of products in a product family by considering the engineering characteristics and by establishing the relationship between customer preferences and product attributes [48]. Fuzzy C-means clustering is applied to classify customer characteristics during the first stage of product definition, which is an essential issue in designing product families from a mass customization perspective [47].

Fuzzy logic has been also applied to issues related to product configuration. Zhu et al. [49] considered uncertain and fuzzy customer requirements by applying fuzzy multiattribute decision making. More recently, this approach has been presented as a method which can be used in the product data management system and on e-commerce websites [50].

According to Fischer et al. [16], interactive product designs are economically and strategically important in the development of new products and processes. In order to be effective and relevant throughout the life cycle of products, the models of products must be scalable and adaptable making necessary the use of different types of analytical techniques such as fuzzy logic, rough sets, and desirability functions to reflect uncertainties, requirements or rules.

2.3 Summary and analysis

Product family design is a challenging process that can be improved in different ways by the use of fuzzy logic [1]. Fuzzy logic allows input information to be given in linguistic terms as colloquially expressed by people. This type of information permits to make better and more accurate decisions due to the wide range of possible answers that can be handled instead of just binary such yes or not.

In this research, we found that fuzzy logic may not yet have been applied to the entire product family design process; it has, however, been used in recent years to improve some specific parts in that process.

This work aims at filling this lack and proposes to exploit the benefits of fuzzy logic to develop a comprehensive methodology to form families of products by integrating all the related topics from a fuzzy logic view instead of partial applications.

3 Methodology to form families of products by applying fuzzy logic

In this section, we propose a methodology to form families of products by applying fuzzy logic. We consider that the use of fuzzy logic can improve the design of product families in a wide range of areas. Fuzzy logic allows opinions, knowledge, and expertise to be provided and managed in linguistic terms



Fig. 1 Methodology to form families of products

commonly used by people. Fuzzy logic is increasingly used in decision aided systems, since it offers many advantages over other traditional decision making techniques. This work is focused on the development and integration of various tools and procedures to form families of products that most closely meet the customer's expectations.

The methodology contains seven steps: (1) market segmentation, (2) generic products configuration, (3) common features identification, (4) modules identification, (5) alternative products configuration, (6) personalized product's configuration, and (7) product variety listing (see Fig. 1). These steps envelop the market considerations, the product family formation, and the product variety consideration.

Step 1. Market segmentation Fuzzy clustering is considered to identify different groups of customers with similar needs and wants. To perform the market segmentation, we propose the following procedure.

1. Selection of product features The most relevant product features $[F_1; F_2...F_n]$ are identified by people with enough experience and knowledge to perform it. For example, we consider that the most relevant features for buying a chair are seat (F_1) , backrest (F_2) , armrest (F_3) , and swivel (F_4) (see Fig. 2).



Fig. 2 Most relevant product features for a chair

- Express customer preferences in linguistic terms The customer preferences for each product feature are evaluated; it could be achieved by a survey. This information is gathered in linguistic terms such as "Very Important" (VI), "Moderately Important" (MI), or "Not Important" (NI). Table 1 shows that the Customer 1 evaluates feature F₁ as very important (VI), feature F₂ as moderately important (MI) and so on.
- 3. *Express customer preferences in numerical terms* In order to apply the fuzzy clustering, customer preferences are translated in numerical terms. For this purpose, we propose to define a (1 to m) scale, in which (m) represents the number of product features being considered. The lowest value from the scale (1) represents the lowest category from the linguistic term, and (m) represents the highest one as depicted in Table 2 based on Table 1 information.
- 4. *Identify the best cluster's scenario* A fuzzy c-means (FCM) clustering iterative method developed by Bezdek

 Table 1
 Customer preferences in linguistic terms

Customer	Product	Product features			
	F ₁	F ₂	F ₃	F ₄	
1	VI	MI	NI	MI	
2	VI	VI	MI	VI	
:	:	÷	÷	÷	
n	VI	NI	MI	VI	

Table 2 Customer preferences in numerical terms

Customer	Product features				
	F ₁	F ₂	F ₃	F ₄	
1	3	2	1	2	
2	3	3	2	3	
:	:	:	:	:	
n	3	1	2	3	

 Table 3
 Membership matrix for a 3-clusters scenario

Cluster	Customers	Customers				
	1	2		n		
1	0.87	0.37		0.32		
2	0.03	0.60		0.08		
3	0.10	0.03		0.60		

[5] is applied for this purpose. According to Xu and Wunsch [45], FCM is one of the most applied. The FCM function starts with an initial guess as to the cluster center. Then, the cluster centers are updated until the satisfaction criteria are met, based on minimizing an objective function, which represents the distance from any given data point to a cluster center. The output is a list of cluster centers and various membership grades for each data point to represent the fuzzy qualities of each cluster.

The cluster membership matrix obtained after the use of FCM with 3 clusters is presented in Table 3. The membership of customer 1 to clusters 1, 2, and 3 are 0.87, 0.03, and 0.10 respectively.

Step 2. Generic product configuration For this configuration, we propose the following procedure adapted from [3].

1. *Consideration of customer preferences* Customer preferences are obtained from the cluster centers in step 1 (see Table 4a) from that we derive the product features expectations (see Table 4b).

 Table 4
 Cluster centers and customer preferences for the 3-clusters scenario

Cluster	F1	F2	 F _m
(a) Cluster c	enters		
1	2.24	2.39	 2.97
2	2.91	2.68	 1.79
3	1.08	1.91	 2.88
(b) Custome	er preferences		
1	2	2	 3
2	3	3	 2
3	1	2	 3

 Table 5
 Prioritization of customer preferences

Linguistic terms	Fuzzy numbers
HI—"Highly Important"	[6 8 10 10]
MI—"Moderately Important"	[3 5 5 7]
NI—"Not Important"	[0 0 2 4]

- 2. *Prioritization of customer preferences* Customer preferences, linguistic values, are translated to fuzzy numbers as presented in Table 5 and Fig. 3.
- 3. *Technical evaluation of product features* This evaluation links the product features contribution to customers' expectations. Figure 4 and Table 6 show how different alternatives for feature 1 contribute to the customer's satisfaction (the highest utility—u) gives more satisfaction.
- 4. *Selection of product features* We use the fuzzy preference relation between A and B, R(A,B), defined in [4] as follows:

$$R(A, B) = [D(A, B) + I(A, B)]/[A(A) + A(B)] \quad (1)$$

Figure 5 shows the different areas form Eq (1) applied to fuzzy numbers A11 and B11.

All the details about the fuzzy preference evaluation of all the possible situations between two normal fuzzy numbers, including trapezoidal, triangular, and rectangular membership functions can be found in [4].

If R(A,B) is equal to 0.5, then fuzzy numbers A and B are indifferent. Considering A is product feature characteristic and B a customer preference for the same feature, this means that a product feature meets exactly the customer preferences.

Step 3. Common feature identification A comparison among all the product configurations is necessary to identify if any alternatives are common between customers' expectations. Let suppose that three generic products have been configured in step 2, one for each cluster in step 1. In Table 7, it is



Fig. 4 Depiction of the

alternatives for feature 1



Table 6 Technical evaluation of product features in fuzzy numbers					
Alternative	F ₁	F ₂		F _m	
1	[0 1 3 6]	[0 4 5 7]	_	[0 1 2 3]	
2	[3 4 6 7]	[8 9 10 10]	-	[1 2 4 5]	
3	[6 8 10 10]	_	_	[2 3 5 6]	
4	_	_	-	[3 4 6 7]	



Fig. 5 Fuzzy number depiction of A₁₁ and B₁₁



Fig. 6 Identification of modules

 Table 7 Common features in generic products

Cluster	Product configuration
1	$\mathbf{F_{13}} = \mathbf{F_{21}} - \mathbf{F_{31}} - \mathbf{F_{43}}$
2	$\mathbf{F_{13}} = \mathbf{F_{22}} - \mathbf{F_{32}} - \mathbf{F_{44}}$
3	$\mathbf{F_{13}} - \mathbf{F_{21}} - \mathbf{F_{32}} - \mathbf{F_{43}}$

possible to note that (F_{13}) which is the first alternative of the product feature 1 is common to the three configurations. F_{13} must be considered as a fixed alternative feature. Common feature identification permits to detect similar preferences among all market segments.

Step 4. Module identification In this paper, we consider a module as the integration of two or more product features. To identify possible modules, we propose the following procedure.

- 1. *Ranking of feature preferences* This is achieved by analyzing all cluster centers with respect to each product feature. The variance among the cluster centers for each product feature is evaluated. The smallest variance will be ranked first and so on.
- 2. Availability of feature alternatives All feature alternatives that are not used in the generic product are considered as alternatives for new product configurations. Let consider that we have different alternatives for each feature. A complete list of these alternatives is presented in Table 8, By pointing the alternatives used in the generic product; it is possible to identify the unused alternatives.

Alternative	F ₁	F ₂	F ₃	F_4
1	[0 1 3 6]	[0 4 5 7]	[0 1 2 3]	[0 1 2 3]
2	[3 4 6 7]	[8 9 10 10]	[1 2 3 4]	[1 2 4 5]
3	[6 8 10 10]	_	[3 4 5 7]	[2356]
4	_	_	[4568]	[3467]

From Table 8, the availability for each feature alternative is: for feature 1 (F_{11} , F_{12}); for feature 3 (F_{33} , F_{34}); and for feature 4 (F_{41} , F_{42}). As can be noted, there is no available alternatives for feature 2.

- 3. *Common features alternative consideration* If there is/are alternative/alternatives which is/are common for all the generic products, then this/these should be included in the module. Let consider the product configurations from step 3 listed in Table 8. For this situation, it is possible to note that F₁₃ is common to all the generic products; therefore, this alternative should be included in all the modules.
- 4. *Module formation* We propose the following considerations to form modules (see Fig. 6).
 - First, the assembly of the module should be started by considering the ranking of the feature preference obtained in phase 1 of this procedure (F₃, F₄, F₂, and F₁).
 - Then, common features should be considered. In this example, F₁₃ alternative should be common to all the modules.
 - And finally, the features with no alternative available cannot be considered. These will be considered in the next step.

Step 5. Alternative product configuration Once the generic products (step 2), common features (step 3), and possible modules (step 4) have been identified, some possible alternative configurations can be identified by applying the following procedure.

- Features with no alternative availability If there exist one or more features with no available alternatives, then all the alternatives for these features will be considered in the alternative product configuration. According to phase 2 in step 4, there is no available alternative for feature 2. That is, F₂₁, and F₂₂ should be part of the new product configurations.
- Alternative product configuration To form alternative configurations of products, the modules identified in step 4 need to be combined with the features with no alternatives' availability as depicted in Fig. 7. Table 9 shows the complete list of the configuration of alternative products.



Fig. 7 Configuration of alternative products

Table 9 List of alternative product configurations

Product alternative formation	Product configuration
$F_{21} + M_1 = P_4$	F_{13} - F_{21} - F_{31} - F_{41}
$F_{21} + M_2 = P_5$	F_{13} - F_{21} - F_{31} - F_{42}
$F_{21} + M_3 = P_6$	F_{13} - F_{21} - F_{32} - F_{41}
$F_{21} + M_4 = P_7$	F_{13} - F_{21} - F_{32} - F_{42}
$F_{22} + M_1 = P_8$	F_{13} - F_{22} - F_{31} - F_{41}
$F_{22} + M_2 = P_9$	F_{13} - F_{22} - F_{31} - F_{42}
$F_{22} + M_3 = P_{10}$	F_{13} - F_{22} - F_{32} - F_{41}
$F_{22} + M_4 = P_{11}$	F_{13} - F_{22} - F_{32} - F_{42}

Table 10 Product features for a particular customer

Alternative	F ₁	F ₂	F ₃	F ₄
1	[0 1 3 6]	[0 4 5 7]	[0 1 2 3]	[0 1 2 3]
2	[3 4 6 7]	[8 9 10 10]	[1 2 3 4]	[1 2 4 5]
3	[6 8 10 10]	_	[3 4 5 7]	[2 3 5 6]
4	-	-	[4 5 6 8]	[3 4 6 7]

Step 6. Personalized product configuration If none of the configured products (generics and alternatives) meet the needs of a specific customer, it is possible to configure a personalized product by performing step 2 considering his/her particular preferences. For example, for a customer who expresses a high preference for all the product features, as can be inferred, the product configuration for this customer should be formed with the highest ranked alternative of each feature. If we consider the information presented in Table 8 of step 4, the product configuration for this particular customer should be: F_{13} - F_{22} - F_{34} - F_{44} (see Table 10).

Step 7. Product variety listing At this point, three types of products: generic, alternative, and personalized form the family of products (see Fig. 8).

Figure 8 shows that one product could belong to different clusters. This situation makes necessary the identification of which product's alternatives are more closely associated to



Fig. 8 Variety of products into a product family

Table 11 Most often preferred features per cluster

Cluster	F ₁	F_2	 F _m
1	MI	MI	 HI
2	HI	HI	 MI
3	NI	MI	 HI



Fig. 9 Identification of product configurations for cluster 2

each cluster. By analyzing the preferences of each cluster, it is possible to identify which products are more convenient for each cluster. Table 11 shows an example of how identify the highest preferred features per each cluster.

Based on the feature preferences for each cluster from Table 11, we may note that P_8 to P_{11} are more closely associated with cluster 2 (see Fig. 9) since these product configurations contain the highest alternatives for features 1 and 2 which are the two more important for customers in cluster 2.

4 Illustrative application

In this section, an illustrative example is presented to show the applicability of the proposed methodology. The formation of a laptop family has been chosen for this purpose.

 Table 12
 Customer feature preferences

Customer	Product features						
	$\overline{F_1}$	F_2	F ₃	F_4	F_5		
1	5	4	3	4	2		
2	1	2	2	3	4		
:	÷	:	÷	÷	÷		
30	5	4	3	3	2		

Step 1. Market segmentation

- 1. Consider product features Let assume that the design team defines the following features as the most relevant considered by the customers at this point of selecting a laptop: processor (F_1) , the operating system (F_2) , the display (F_3) , the memory (F_4) , and the hard drive (F_5) .
- Express customer preferences in linguistic terms Let consider that a group of thirty customers has been surveyed about their preferences with respect to each feature. The customer preferences for each feature are expressed in the linguistic terms: "highly important" (HI), "important" (I), "moderately important" (MI), "somewhat important" (SI), and "not important" (NI).
- 3. *Express customer preferences in numerical terms* To apply fuzzy clustering, it is necessary to translate the customer preferences in numerical terms. We use a range from 5 to 1, where 5 represents "highly important", 4 "important", and so on. Table 12 lists a part of these preferences. The full list is presented in Table 21.
- 4. *Identify the best cluster's scenario* By using the fuzzy logic toolbox of Matlab, the fuzzy clustering has been performed. Three different scenarios have been evaluated: (a) four clusters, (b) three clusters, and (c) two clusters. The analysis of the membership matrix and the cluster centers are used to determine the best scenario as follows.
 - *Membership matrix analysis* A portion of the obtained membership matrix is presented in Fig 10. For the case (a), it is possible to note that the membership of customer 1 to cluster 4, 3, 2, and 1 is 0.89, 0.08, 0.02, and 0.01 respectively.

Step 2. Generic product configuration Let apply the following procedure to configure generic products.

 Consideration of customer preferences The customer preferences are obtained by analyzing the cluster centers. Figure 11(a) lists the cluster center for the 3-cluster sce-

Cluster	Customers					
Cluster	1	2		30		
1	0.01	0.36		0.02		
2	0.02	0.59		0.03		
3	0.08	0.03		0.42		
4	0.89	0.03		0.53		

Cluster	Customers				
Cluster	1	2		30	
1	0.09	0.27		0.02	
2	0.88	0.04		0.97	
3	0.04	0.69		0.01	

(b) Three clusters

Cluster	Customers				
Cluster	1	2		30	
1	0.95	0.03		0.99	
2	0.05	0.97		0.01	

(c) Two clusters

(a) Four clusters

Fig. 10 Membership matrix for each scenario

Fig. 11 Cluster centers and customer preferences for the 3-cluster scenario

Cluster	F_1	F ₂	F ₃	F_4	F ₅
1	2.34	2.36	2.64	3.03	2.95
2	4.91	4.18	2.85	3.05	1.57
3	1.08	1.98	2.61	3.70	4.70

Cluster F_1 F_2 F_3 F_4 F_5 1 SI SI MI MI MI 2 HI Ι MI MI SI

(a) Cluster centers

(b) Customer preferences

MI

SI

 Table 13
 General prioritization of customer preferences

Linguistic terms	Fuzzy numbers
HI—"Highly Important"	[7 9 10 10]
I—"Important"	[5689]
M—"Moderately Important"	[3 5 5 7]
SI—"Somewhat Important"	[1 2 4 5]
NI—"Not Important"	[0 0 1 3]

nario. This information is expressed in linguistic terms (see Fig. 11b).

Figure 11 shows that the cluster 1 includes customers moderately interested in almost all the laptop features. Cluster 2 includes customers more interested in features such as the processor (F_1), and moderately interested in display (F_3), and memory (F_4). Cluster 3 includes the customers who are more interested in storage capacity (F_5).

- 2. *General prioritization of customer preferences* Let assume that a team of specialists defined the following general prioritization of customer preferences as depicted in Table 13.
- 3. *Technical evaluation of product features* Let use trapezoidal fuzzy numbers to represent the technical evaluation of each feature alternative (see Table 14).
- 4. Selection of product features. Let analyze the fuzzy preference relation $R(F_{ij}, C_{ki})$ between product features and customer preference.
 - Analysis of fuzzy preference relations In this application, the fuzzy preference relation is denoted as R(F_{ij}, C_{ki}), where F_{ij}={F₁₁, F₁₂, ..., F_{nm}} is the set of the evaluations of the feature (i) for each feature alterna-

HI

I

261

 Table 14
 Technical evaluation of product features represented by fuzzy numbers

NI

3

mannoero				
F ₁	F ₂	F ₃	F ₄	F ₅
[0 1 4 6]	[0 4 5 7]	[0 1 2 3]	[0 2 4 6]	[0 1 2 3]
[2468]	[8 9 10 10]	[1 2 3 4]	[2367]	[1 2 4 5]
[7 8 10 10]	-	[3 4 5 7]	[4 6 7 9]	[2356]
-	-	[4 5 6 8]	[7 8 10 10]	[3 4 6 7]
-	-	[6789]	-	[5689]
-	-	[7 8 10 10]	-	[7 8 10 10]

tive (j) for all i=1, 2,..., n, and for all j=1, 2,..., m, and $C_{ki} = \{C_1, C_2, ..., C_{pn}\}$ is the set of customer preferences of a cluster (k) for each feature (i) for all k=1, 2,..., p.

Table 15 lists the fuzzy preference relation for all the relations in cluster 1. Tables 22, 23 present these preferences for cluster 2 and cluster 3 respectively.

To identify the best alternative of each feature for each cluster, we consider that the $R(F_{ij}, C_{ki})$ nearest to 0.5 corresponds to the alternative that should be part of the generic product (see Table 16).

Based on the previous statement, the product configuration for each cluster is: $F_{11}-F_{21}-F_{33}-F_{42}-F_{54}$ for cluster 1, $F_{13}-F_{22}-F_{33}-F_{42}-F_{52}$ for cluster 2, and $F_{11}-F_{21}-F_{33}-F_{43}-F_{56}$ for cluster 3.

Step 3. Common feature identification Table 17 permits to identify that F_{33} is common to all the generic products for all the clusters. That is to say; medium-sized laptop displays are being preferred by most of the customers. This alternative will now be considered fixed for all configurations.

Table 15 Fuzzy preference relation of cluster 1

$F_{ij} \backslash C_{ki}$	C ₁₁	C ₁₂	C ₁₃	C ₁₄	C ₁₅
	[1 2 4 5]	[1 2 4 5]	[3 5 5 7]	[3 5 5 7]	[3 5 5 7]
F ₁₁ [0 1 4 6]	0.4667				
F ₁₂ [2 4 6 8]	0.7857				
F ₁₃ [7 8 10 10]	1.0000				
F ₂₁ [0 4 5 7]		0.6429			
F ₂₂ [8 9 10 10]		1.0000			
F ₃₁ [0 1 2 3]			0.0000		
F ₃₂ [1 2 3 4]			0.0000		
F ₃₃ [3 4 5 7]			0.4444		
F ₃₄ [4 5 6 8]			0.6667		
F35 [6789]			0.9670		
F ₃₆ [7 8 10 10]			1.0000		
F ₄₁ [0 2 4 6]				0.1875	
F ₄₂ [2 3 6 7]				0.4167	
F ₄₃ [4 6 7 9]				0.7750	
F ₄₄ [7 8 10 10]				1.0000	
F ₅₁ [0 1 2 3]					0.0000
F ₅₂ [1 2 4 5]					0.0000
F ₅₃ [2 3 5 6]					0.3000
F ₅₄ [3 4 6 7]					0.5000
F ₅₅ [5689]					0.8666
F ₅₆ [7 8 10 10]					1.0000

Step 4. Module identification To identify possible modules, let apply the following procedure.

- 1. *Ranking of feature preferences* The order is obtained by calculating the variance among the cluster centers for each product feature. The feature with the smallest variance will be the first in the ranking. Based on the information in Table 18, the feature ranking is F₃, F₄, F₂, F₅, and F₁.
- 2. Availability of feature alternatives Table 16 permits to identify feature alternatives not used in the generic product configurations. The availability for each feature alternative is: for feature 1 (F₁₂); for feature 3 (F₃₁, F₃₂, F₃₄, F₃₅, and F₃₆); for feature 4 (F₄₁ and F₄₄); and for feature 5 (F₅₁, F₅₃, and F₅₅). There are no available alternatives for feature 2.
- 3. *Common features alternative consideration* From Table 16, it is possible to identify that F₃₃ is common to product for each cluster. Therefore, it should be included in all the modules.
- 4. *Module formation* Let follows the following considerations to form modules. Figure 12 depicts this procedure.
 - The assembly of the module should be started by considering the ranking of the feature preference

Table 16 Produ	act features for each	cluster				
Features	Clusters					
	1	2	3			
F ₁₁	0.0333	0.5	0.2692			
F ₁₂	0.2857	0.4792	0.4792			
F ₁₃	0.5	0.0556	0.5			
F ₂₁	0.1429	0.4048	0.1429			
F ₂₂	0.5	0.1667	0.5			
F ₃₁	0.5	0.5	0.5			
F ₃₂	0.5	0.5	0.5			
F ₃₃	0.0556	0.0556	0.0556			
F ₃₄	0.1667	0.1667	0.1667			
F ₃₅	0.467	0.4167	0.4167			
F ₃₆	0.5	0.5	0.5			
F41	0.3125	0.3125	0.5			
F ₄₂	0.0833	0.0833	0.3571			
F43	0.275	0.2750	0.0833			
F44	0.5	0.5	0.3182			
F51	0.5	0.3	0.5			
F52	0.5	0	0.5			
F ₅₃	0.2	0.1667	0.5			
F54	0	0.3333	0.5			
F ₅₅	0.3666	0.5	0.3667			
F56	0.5	0.5	0.0556			

 Table 17
 Product configuration for each cluster

Cluster	Product configuration		
1	$F_{11} - F_{21} - F_{33} - F_{42} - F_{54}$		
2	$F_{13} - F_{22} - F_{33} - F_{42} - F_{52}$		
3	$F_{11} - F_{21} - F_{33} - F_{43} - F_{56}$		

Table	18	Analysi	s of	cluster
centers	wit	h respe	ct to p	product
features	3			

Feature	Variance
1	3.82
2	1.39
3	0.02
4	0.15
5	2.46
2 3 4 5	1.39 0.02 0.15 2.46

obtained in phase 1 of this procedure $(F_3, F_4, F_2, F_5, and F_1)$.

• Then, common features should be considered. In this case, F₃₃ should be common to all the modules.



Fig. 12 Module identification

• And finally, features with no alternative's availability cannot be considered. These features will be considered in the next step.

Step 5. Alternative product configuration

- Features with no alternative availability In step 4, we found that there are no available alternatives for feature
 That is; F₂₁ and F₂₂ will be part of the new product configuration.
- 2. Alternative product configuration This configuration is the combination of the formed modules with the alternatives of the features with no availability (see Fig. 13). Table 19 lists these alternative configurations.

Step 6. Personalized product configuration If a particular customer is not satisfied with the customized

products offered. A personalized product could be designed. Let assume that one customer wants his product to be person-

Table 19 Features of the alternative product configuration

Product alternative formation	Product configuration
$F_{21} + M_1 = P_4$	F ₁₂ -F ₂₁ -F ₃₃ -F ₄₁ -F ₅₁
$F_{21} + M_2 = P_5$	F_{12} - F_{21} - F_{33} - F_{41} - F_{53}
$F_{21} + M_3 = P_6$	F_{12} - F_{21} - F_{33} - F_{41} - F_{55}
$F_{21} + M_4 = P_7$	F_{12} - F_{21} - F_{33} - F_{44} - F_{51}
$F_{21} + M_5 = P_8$	F_{12} - F_{21} - F_{33} - F_{44} - F_{53}
$F_{21} + M_6 = P_9$	F_{12} - F_{21} - F_{33} - F_{44} - F_{55}
$F_{22} + M_1 = P_{10}$	F_{12} - F_{22} - F_{33} - F_{41} - F_{51}
$F_{22} + M_2 = P_{11}$	F_{12} - F_{22} - F_{33} - F_{41} - F_{53}
$F_{22} + M_3 = P_{12}$	F_{12} - F_{22} - F_{33} - F_{41} - F_{55}
$F_{22} + M_4 = P_{13}$	F_{12} - F_{22} - F_{33} - F_{44} - F_{51}
$F_{22} + M_5 = P_{14}$	F_{12} - F_{22} - F_{33} - F_{44} - F_{53}
$F_{22} + M_6 = P_{15}$	F_{12} - F_{22} - F_{33} - F_{44} - F_{55}



Fig. 14 Types of product configurations in the family of products

alized, and for him/her, all the product features are "highly important" (HI). Table 24 lists the full fuzzy preference relation for this case. Based on Table 18 the product configuration for this particular customer should be: F_{13} - F_{22} - F_{36} - F_{44} - F_{56} .



Fig. 13 Alternative product configuration



Fig. 15 Identification of product configuration for each cluster

Table 20 Most often preferred features per cluster

Cluster	F ₁	F ₂	F ₃	F ₄	F ₅
1	SI	SI	MI	MI	MI
2	HI	Ι	MI	MI	SI
3	NI	SI	MI	Ι	HI

Step 7. Product variety listing Figure 14 lists the different product configurations (generic, modular, and personalized) contained in the product family.

From Fig. 15 products 1, 2, and 3 belong to clusters 1, 2, and 3 respectively. With respect to products 4 to 15, it is important to identify which of these modular customized products are more closely associated to each cluster. By analyzing the customer preferences of each cluster, it is possible to identify the features most often preferred in each cluster (see Table 20).

Figure 15 shows that P_4 to P_9 are more closely associated with cluster 1, P_{10} to P_{15} with cluster 2, and P_7 to P_9 and P_{13} to P_{15} with cluster 3.

5 Conclusions

A detailed methodology to form families of products using fuzzy logic has been proposed in this work. The integration of fuzzy logic is the result of the development, adaptation, and improvement of new applications. Fuzzy logic has been chosen to improve the decision making process in the methodology. It is aimed at contributing to increase customers' satisfaction by making better decisions to meet their preferences. The output of this methodology is a family of products, which is formed by three types of products: generic products for each segment of the market, modular customized products associated with each segment of the market, and personalized product for a specific customer. This methodology contributes to the possibility of offering both generic and standardized products for different segments, and to reduce the costs of the product due to standardization of components and associated processes. Some future research directions could include the analysis and consideration of component-level instead of a feature-level along the stages and steps of the methodology.

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Appendix

See Tables 21, 22, 23, 24.

 Table 21
 Customer preferences for each product feature

 C_{22}

 C_{23}

 C_{24}

 C_{21}

 $F_{ij} \backslash C_{ki}$

Customer	Product features					
	$\overline{F_1}$	F_2	F ₃	F_4	F ₅	
1	5	4	3	4	2	
2	1	2	2	3	4	
3	4	3	2	3	2	
4	1	2	3	4	5	
5	5	5	3	4	1	
6	5	4	3	3	2	
7	4	4	3	5	2	
8	2	2	2	3	4	
9	5	4	3	2	1	
10	5	4	2	2	2	
11	1	3	3	3	4	
12	2	2	3	3	3	
13	1	1	3	4	5	
14	2	3	2	3	4	
15	1	3	3	3	5	
16	5	4	3	2	1	
17	5	4	3	3	2	
18	1	2	3	4	4	
19	2	2	3	3	3	
20	5	4	3	3	1	
21	3	3	2	3	2	
22	5	5	3	4	1	
23	1	2	2	2	5	
24	5	5	3	4	2	
25	1	2	2	4	5	
26	1	1	2	5	5	
27	3	2	3	3	2	
28	5	4	2	1	1	
29	1	2	3	4	5	
30	5	4	3	3	2	

	[7 9 10 10] [5689	9] [3557	7] [3557	[1 2 4 5]
F ₁₁ [0 1 4 6]	0.0000				
F ₁₂ [2 4 6 8]	0.0208				
F ₁₃ [7 8 10 10]	0.4444				
F ₂₁ [0 4 5 7]		0.0952			
F ₂₂ [8 9 10 10]		0.9444			
F ₃₁ [0 1 2 3]			0.0000		
F_{32} [1 2 3 4]			0.0000		
F ₃₃ [3 4 5 7]			0.4444		
F ₃₄ [4 5 6 8]			0.6667		
F ₃₅ [6 7 8 9]			0.9167		
F ₃₆ [7 8 10 10]			1.0000		
$F_{41} [0 2 4 6]$				0.1875	
$F_{42} [2367]$				0.4167	
F_{42} [4 6 7 9]				0.7750	
F ₄₄ [7 8 10 10]				1.0000	
$F_{51} [0 2 3]$				1.0000	0.2000
$F_{52} [1 2 4 5]$					0.5000
$F_{52}[2356]$					0.6667
$F_{53}[2550]$					0.8333
$F_{54} [5 + 6 7]$					1 0000
$F_{55} [5089]$					1.0000
					1.0000
Table 23 Fuzzy	y preferenc	e relation	of cluster 3		
$F_{ij} \backslash C_{ki}$	C ₃₁	C ₃₂	C ₃₃	C ₃₄	C ₃₅
	[0 0 1 3]	[1 2 4 5]	[3 5 5 7]	[5689]	[7 9 10 10]
	[···]	r - 1		[· · · ·]	[····]
F ₁₁ [0 1 4 6]	0.7692	,		[]	
F ₁₁ [0 1 4 6] F ₁₂ [2 4 6 8]	0.7692 0.9792			[]	
$F_{11} [0 1 4 6] F_{12} [2 4 6 8] F_{13} [7 8 10 10]$	0.7692 0.9792 1.0000				
$F_{11} [0 1 4 6]$ $F_{12} [2 4 6 8]$ $F_{13} [7 8 10 10]$ $F_{21} [0 4 5 7]$	0.7692 0.9792 1.0000	0.6429		[]	[]
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429			
$ F_{11} [0 1 4 6] \\ F_{12} [2 4 6 8] \\ F_{13} [7 8 10 10] \\ F_{21} [0 4 5 7] \\ F_{22} [8 9 10 10] \\ F_{31} [0 1 2 3] $	0.7692 0.9792 1.0000	0.6429	0.0000		
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000		
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444		
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667		
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167		
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{33} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000		
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000	
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429	
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 5 \ 7 \ 9 \right] \\ F_{43} \left[4 \ 5 \ 7 \ 9 \right] \\ \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167	
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \ 9 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \ 10 \right] \\ F_{45} \left[7 \ 8 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 $	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ \end{array}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \ 5 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000 0.0000 0.0000
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{53} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{53} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{53} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{54} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{54} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \ 7 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \ 7 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \ 7 \right] \\ F_{55} \left[2 \ 3 \ 5 \ 6 \ 7 \right] \\ F_{55} \left[2 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \ 5 \$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000 0.0000 0.0000 0.0000
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{53} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{54} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{54} \left[3 \ 4 \ 6 \ 7 \right] \\ F_{56} \left[5 \ 4 \ 7 \right] \\ F_{56} \left[5 \ 4 \ 7 \right] \\ F_{56} \left[5 \ 4 \ 7 \right] \\ F_{56} \left[5 \ 6 \ 7 \right] \\ F_{56} \left[5 \ 7 \right] \ \ F_{56}$	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000 0.0000 0.0000 0.0000 0.0000
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{53} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{54} \left[3 \ 4 \ 6 \ 7 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \ 10 \right] \\ F_{55} \left[7 \ 8 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 $	0.7692 0.9792 1.0000	0.6429	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000 0.0000 0.0000 0.1333 0.4444
$\begin{array}{c} F_{11} \left[0 \ 1 \ 4 \ 6 \right] \\ F_{12} \left[2 \ 4 \ 6 \ 8 \right] \\ F_{13} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{21} \left[0 \ 4 \ 5 \ 7 \right] \\ F_{22} \left[8 \ 9 \ 10 \ 10 \right] \\ F_{31} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{32} \left[1 \ 2 \ 3 \ 4 \right] \\ F_{33} \left[3 \ 4 \ 5 \ 7 \right] \\ F_{34} \left[4 \ 5 \ 6 \ 8 \right] \\ F_{35} \left[6 \ 7 \ 8 \ 9 \right] \\ F_{36} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{41} \left[0 \ 2 \ 4 \ 6 \right] \\ F_{42} \left[2 \ 3 \ 6 \ 7 \right] \\ F_{43} \left[4 \ 6 \ 7 \ 9 \right] \\ F_{44} \left[7 \ 8 \ 10 \ 10 \right] \\ F_{51} \left[0 \ 1 \ 2 \ 3 \right] \\ F_{52} \left[1 \ 2 \ 4 \ 5 \right] \\ F_{53} \left[2 \ 3 \ 5 \ 6 \right] \\ F_{54} \left[3 \ 4 \ 6 \ 7 \right] \\ F_{55} \left[5 \ 6 \ 8 \ 9 \right] \\ F_{56} \left[7 \ 8 \ 10 \ 10 \right] \\ \end{array}$	0.7692 0.9792 1.0000	0.6429 1.0000	0.0000 0.0000 0.4444 0.6667 0.9167 1.0000	0.0000 0.1429 0.4167 0.8182	0.0000 0.0000 0.0000 0.0000 0.1333 0.4444

C₂₅

Table 24 Fuzzy preference relation of customer X

	. 1				
$F_{ij} \setminus C_{ki}$	C _{x1}	C _{x2}	C _{x3}	C _{x4}	C _{x5}
	[7 9 10 10)][7 9 10	10][7 9 10 1	10][7 9 10	10][7 9 10 10]
F ₁₁ [0 1 4 6]	0.0000				
F_{12} [2 4 6 8]	0.0208				
F ₁₃ [7 8 10 10]0.4444				
$F_{21} [0 4 5 7]$		0.0000			
F ₂₂ [8 9 10 10]	0.5714			
F ₃₁ [0 1 2 3]			0.0000		
F ₃₂ [1 2 3 4]			0.0000		
F ₃₃ [3 4 5 7]			0.0000		
F ₃₄ [4 5 6 8]			0.0000		
F ₃₅ [6789]			0.1667		
F ₃₆ [7 8 10 10]		0.4444		
F ₄₁ [0 2 4 6]				0.0000	
F ₄₂ [2 3 6 7]				0.0000	
F ₄₃ [4 6 7 9]				0.1000	
F ₄₄ [7 8 10 10]			0.4444	
F ₅₁ [0 1 2 3]					0.0000
F ₅₂ [1 2 4 5]					0.0000
F ₅₃ [2 3 5 6]					0.0000
F ₅₄ [3 4 6 7]					0.0000
F ₅₅ [5689]					0.1333
F ₅₆ [7 8 10 10]				0.4444

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