IMPACTS OF MINIMUM ACTIVITY LEVEL AND MULTI-SOURCING ON PRODUCT FAMILY AND SUPPLY CHAIN DESIGN

Bertrand Baud-Lavigne†‡, Bruno Agard‡ and Bernard Penz†

† Université de Grenoble / Grenoble INP / UJF Grenoble 1 / CNRS
G-SCOP UMR5272 Grenoble, F-38031, France

‡ CIRRELT, Département de Mathématiques et Génie Industriel
École Polytechnique de Montréal
C.P. 6079, succ. Centre-Ville, Montréal (Québec), H3C 3A7, Canada

Abstract
This study considers the problem of joint design of the supply chain structure and the product family bill-of-materials through substitution decisions: standardization, externalization and alternative operating sequence. The aim of this paper is to analyze the influence of some strategical decisions (minimum activity level and multi-sourcing) on the optimal design of the product family and its supply chain. In order to avoid mass lay-offs that have major economical and image impact, companies may be constraint to keep production in centers where the cost is high, leading them to produce with a sub-optimal solution considering production costs. In this context, the following issues are tested and discussed: what are the consequences of the obligation to keep a minimum activity level on a center? Which types of operation are kept ? Does multi-sourcing lead to increase standardization ? How does it affect product allocation in the supply chain ? Experiments are led on a case study with several production centers around the world.

Keywords: Supply chain, Product substitution, Multi-sourcing, Minimum activity, Mixed Integer Linear Programming

1 Introduction

The internationalization of the markets compels most companies to diversify their offers to meet customer's demands. This diversity affects conception, production and distribution processes in decreasing economies of scale. In this condition, providing a wide variety of products that meets customer's needs while controlling production and logistical costs is of major importance. Simultaneous design of the supply chain and the product family can help to optimize these costs.

Works on product family design usually take into account production and logistical constraints. Mass customization [9] is now a well spread conception technique to achieve large variety of product designs at minimum cost. This allows focusing on the optimization of the product family, considering that assembly constraints are more flexible. Many problematics arise as determining the product family variety [3] or the module optimization [1] to minimize production costs.

However, product and supply chain optimization have not been tackled simultaneously until recent years. The need of joint optimization has been highlight by [2] and [4], showing that both decisions have impacts on each other. [2] focuses on a production network and compares sequential design with simultaneous design on a case study and with an analytic analysis. Considering explicit bill-of-materials (BOM) in supply chain design model is a recent field that is yet little studied. A single-period, multi-product and multi-level models has been proposed by [8], and a multi-period model has been presented by [10]. In these models, BOM is fixed.

Very few studies optimize simultaneously the product and the supply chain. Two approaches are listed in the literature. The first approach seeks to define the best product family which meets the market needs, by using generic BOM to model the product part of the problem ([7], [12]). In these formulations, BOM are determined so as to respect assembly constraints. The second approach considers the final products as fixed, but the BOM are more-or-less flexible. To model this in an assembly-to-order context where the final assembly time is constrained, [5] considered function and modular design, in which all the assemblies are possible. [6] defined several alternative BOM, one being selected in the optimal solution. This approach needs a complete enumeration of all product configurations. In return, both formulation and solution are facilitated. The only fully integrated model to our knowledge has been proposed by Chen in 2010 [4].

The aim of this paper is to analyze the impact of strategical decisions widely spread in industry that could redefine the optimal supply chain. Section 2 describes the aim of the model used for the experiments, its mathematical formulation and some limitations. Concepts of multi-sourcing and minimum activity are explained and experimented in Section 3. Section 4 concludes the paper and gives some perspectives.

2 An optimization model for joint product and supply chain design

The model used in this study defines simultaneously the supply chain structure and the bill-of-materials (BOM) of the product family. The supply chain has four layers: suppliers, production centers, distribution centers and customers. Product families are represented through the BOM of each product which shares some common sub-assemblies and components. By definition, all assemblies can be produced and transported from a center to another. The aim of the optimization is to determine:

1. the bill-of-materials composition: selection of the relevant components and sub-assemblies,
2. the suppliers selection / production centers location,
3. from where each component is supplied,
4. which workstation are allocated in each production center,
5. the allocation of each assembly to each production center.

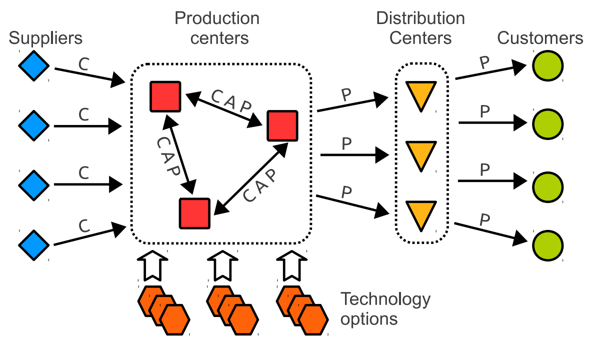

Figure 1: Schematic supply chain network.

Products and supply chain are illustrated in Figure 1 and 2. Figure 1 describes an example of a supply chain with four suppliers, three production centers, three distribution centers and four customers. Flow contents are only components between the suppliers and the plants, products between the plants and the customers and all types of part between the plants. Three types of options can be allocated to each production center.
The example in Figure 2a considers two products: $P1$ and $P2$. Each product is composed of two sub-assemblies. $P1$ is composed of sub-assemblies A and B, while $P2$ contains A' and B'. Sub-assemblies are composed of components 1 to 5.
Substitution possibilities can be express through explicit equivalences between assemblies. Three types of substitution are considered:

standardization: a component (or sub-assembly) can be upgraded by another one with more functionalities or with a better quality. In one hand, individual part is more expensive to buy, to produce or to transport, so variable costs increase. On the other hand, diversity decreases and allows better economy of scale. An illustration is provided in Figure 2b. Sub-assembly A' can be replaced by A, B and B' can be replaced by a new sub-assembly B'',

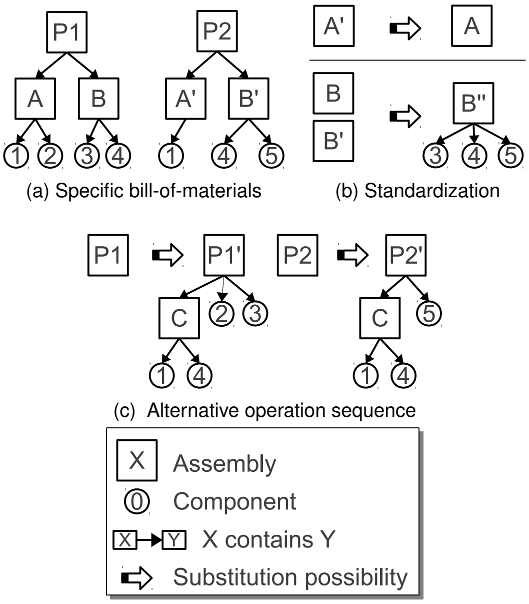

Figure 2: Bill-of-materials and substitution illustrations.

externalization: a sub-assembly is bought directly from a subcontractor. Fixed costs are nearly avoid while increasing variable costs. In the example on Figure 2a, the sub-assembly A or B could be replaced by a component bought from a supplier,

alternative operating sequence: other possible assembling order can permit better commonality without necessarily changing costs. Figure 2c presents a sequence where sub-assembly C has a good commonality without adding extra function.

This paper focuses on standardization, although these substitutions options can be expressed by the model described in the following section.
The concept of substitution has been reconsidered in order to model the problem with a reasonable amount of linear constraints. The typical way to express substitution would be to use the BOM as a decision variable. In our understanding, this would conduct to quadratic constraints between BOM and production decision variables in most supply chain design models. Another way proposed in [4] is to use decision variables to express exactly which quantity of each alternative is used for each produced assembly produced. This leads to a considerable amount of decision variables. Our model described below has a simplified approach: product substitution is considered through product transformation. When part X is able to substitute part Y, a virtual process can transform X into Y. Then, quantity of Y on a plant is mixed between actual Y part and alternatives that have been transformed in Y. This view of the problem allows substitution while keeping a light formulation. Indeed, the number of additional variables is exactly the substitution possibilities.
The problem is modelled with flow and fixed cost constraints. Substitution possibilities are included at each level of the BOM (components, sub-assemblies and final product). The supply chain and the product family are optimized simultaneously.
Sets:

- $\mathcal{P}$: products ; index: $p, q \in \mathcal{P}$
  - $\mathcal{R} \subset \mathcal{P}$: raw materials or components,
  - $\mathcal{M} \subset \mathcal{P}$: manufactured products / sub-assemblies,
  - $\mathcal{F} \subset \mathcal{P}$: finished products.
- $\mathcal{N}$: network nodes ; index: $i, j \in \mathcal{N}$
  - $\mathcal{S} \subset \mathcal{N}$: suppliers,
  - $\mathcal{U} \subset \mathcal{N}$: production centers,
  - $\mathcal{D} \subset \mathcal{N}$: distribution centers,
  - $\mathcal{C} \subset \mathcal{N}$: customers.
- $\mathcal{T}$: technologies ; index: $t \in \mathcal{T}$
- $\mathcal{O}$: capacity options ; index: $o \in \mathcal{O}$

Parameters:

- g^{pq}: quantity of q in p. q can be a component or a sub-assembly. g represents the bill-of-materials, $p \in \mathcal{M} \cup \mathcal{F}, q \in \mathcal{R} \cup \mathcal{M}$,
- d_i^p: demand of product p by customer i, $p \in \mathcal{M}, i \in \mathcal{C}$,
- c^o: option o capacity, $o \in \mathcal{O}$,
- l^{pt}: processing time of product p on technology t, $p \in \mathcal{M} \cup \mathcal{F}, t \in \mathcal{T}$,
- l^p: labour time of product p, $p \in \mathcal{M} \cup \mathcal{F}$,
- $\mathcal{T}^p \subset \mathcal{T}$: technologies needed by product p, $p \in \mathcal{M} \cup \mathcal{F}$,
- $\mathcal{P}^p \subset \mathcal{P}$: products that can substitute p, $p \in \mathcal{P}$.

The decision variables are presented in table 1. Each variable is associated with its proper cost.

Table 1: Decision variables (DV) and their associated costs.

	Domain	DV	Cost
Quantity of p produced on i	$\mathbb{N}$	A_i^p	α_i^p
Production of p on i	$\{0,1\}$	B_i^p	β_i^p
Quantity of p that substitute q on i	$\mathbb{N}$	S_i^{pq}	σ_i^{pq}
Flow of p between i and j	$\mathbb{N}$	F_{ij}^p	ϕ_{ij}^p
Use of flow of p between i and j	$\{0,1\}$	T_{ij}^p	τ_{ij}^p
Use of axis between i and j	$\{0,1\}$	L_{ij}	λ_{ij}
Number of option o on i	$\mathbb{N}$	O_i^l	ω_i^l
Use of node i	$\{0,1\}$	Z_i	ζ_i
Production of p	$\{0,1\}$	M^p	

The mathematical model is as follows.

$$\begin{aligned} Z = min \quad & \sum_{i\in\mathcal{N}}\sum_{p\in\mathcal{P}}(A_i^p\alpha_i^p + B_i^p\beta_i^p) \\ + \quad & \sum_{i\in\mathcal{N}}\sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{P}^p} S_i^{qp}\sigma_i^{qp} \\ + \quad & \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{i\}}\sum_{p\in\mathcal{P}}(F_{ij}^p\phi_{ij}^p + T_{ij}^p\tau_{ij}^p) \\ + \quad & \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{i\}} L_{ij}\lambda_{ij} \\ + \quad & \sum_{i\in\mathcal{N}}\sum_{o\in\mathcal{O}} O_i^o\omega_i^o \\ + \quad & \sum_{i\in\mathcal{N}} Z_i\zeta_i \end{aligned} \tag{1}$$

The objective function (1) minimizes procurement, production and transportation fixed and variable costs.

Constraints (2) to (6) are flow constraints. The sources are the component flows from the suppliers to the plants, the sinks are final product flows to customers. Constraint (2) considers the flow of each manufactured assembly on each production center.

$$\begin{aligned} A_i^p \quad & + \sum_{j\in\mathcal{U}\setminus\{i\}} F_{ji}^p + \sum_{q\in\mathcal{P}^p} S_i^{qp} \\ & = \sum_{j\in\mathcal{U}\setminus\{i\}} F_{ij}^p + \sum_{q\in\mathcal{M}\cup\mathcal{F}} g^{qp}A_i^q + \sum_{q/p\in\mathcal{P}^q} S_i^{pq} \\ & \forall i \in \mathcal{U}, \forall p \in \mathcal{M} \end{aligned} \tag{2}$$

Constraint (3) considers the flow of each component on each production center.

$$\begin{aligned} & \sum_{j\in(\mathcal{S}\cup\mathcal{U})\setminus\{i\}} F_{ji}^p + \sum_{q\in\mathcal{P}^p} S_i^{qp} \\ & = \sum_{j\in\mathcal{U}\setminus\{i\}} F_{ij}^p + \sum_{q\in\mathcal{M}\cup\mathcal{F}} g^{qp}A_i^q + \sum_{q/p\in\mathcal{P}^q} S_i^{pq} \\ & \forall i \in \mathcal{U}, \forall p \in \mathcal{R} \end{aligned} \tag{3}$$

Constraint (4) considers the flow of each component on each supplier.

$$A_i^p = \sum_{j\in\mathcal{U}} F_{ij}^p \qquad \forall i \in \mathcal{S}, \forall p \in \mathcal{R} \tag{4}$$

Constraint (5) considers the flow of each final product on each distribution center.

$$\sum_{j\in\mathcal{U}\cup\mathcal{D}\setminus\{i\}} F_{ji}^p = \sum_{j\in\mathcal{D}\cup\mathcal{C}\setminus\{i\}} F_{ij}^p \qquad \forall i \in \mathcal{D}, \forall p \in \mathcal{F} \tag{5}$$

Constraint (6) considers the flow of each final product on each production center.

$$A_i^p + \sum_{j\in\mathcal{U}} F_{ji}^p = \sum_{j\in\mathcal{D}\cup\mathcal{C}\setminus\{i\}} F_{ij}^p \qquad \forall i \in \mathcal{U}, \forall p \in \mathcal{F} \tag{6}$$

Constraint (7) assures that customer's demands are satisfied.

$$\sum_{j\in\mathcal{D}} F_{ij}^p + \sum_{q\in\mathcal{P}^p} S_i^{qp} = \sum_{q/p\in\mathcal{P}^q} S_i^{pq} + d_i^p \qquad \forall i \in \mathcal{C}, \forall p \in \mathcal{F} \tag{7}$$

Constraint (8) assures that fixed costs are paid when a component is provided by a supplier or when an assembly is manufactured on a center. A_{max}^p is the upper bound of $A_i^p \quad \forall i \in U$.

$$A_i^p \leq B_i^p A_{max}^p \qquad \forall i \in \mathcal{S} \cup \mathcal{U} \cup \mathcal{D}, \forall p \in \mathcal{P} \tag{8}$$

Constraint (9) assures that fixed costs are paid when a supplier or a center is used.

$$B_i^p \leq Z_i \qquad \forall i \in \mathcal{S} \cup \mathcal{U} \cup \mathcal{D}, \forall p \in \mathcal{P} \tag{9}$$

Constraint (10) defines the number of workstation needed on a center.

$$\sum_{p/u\in\mathcal{P}^p} l^{pt}A_i^p \leq \sum_{o\in\mathcal{O}^t} O_i^o c^o \qquad \forall i \in \mathcal{U}, \forall t \in \mathcal{T} \tag{10}$$

Constraint (11) assures that fixed costs are paid when a product is transported between two nodes.

$$F_{ij}^p \leq T_{ij}^p A_{max}^p \qquad \forall i \in \mathcal{N}, \forall j \in \mathcal{N} \backslash \{i\}, \forall p \in \mathcal{P} \quad (11)$$

Constraint (12) assures that fixed costs are paid when a link between two nodes is used.

$$T_{ij}^p \leq L_{ij} \qquad \forall i \in \mathcal{N}, \forall j \in \mathcal{N} \setminus \{i\}, \forall p \in \mathcal{P} \qquad (12)$$

Constraint (13) limits substituted products to be used within the center they has been created.

$$\sum_{q \in \mathcal{P}^p} S_i^{qp} \leq \sum_{q \in \mathcal{M} \backslash p} g^{qp} A_i^q + \sum_{j \in \mathcal{C}} F_{ij}^p \qquad \forall i \in \mathcal{U}, \forall p \in \mathcal{P} \qquad (13)$$

3 Experiments

3.1 Design of experiment

Experiments are led by solving the MILP presented in Section 2 extended with additional constraints presented below. ILOG CPLEX 12 Java libraries are used on a laptop under 64 bits OS with a 2.26 GHz Intel Core 2 Duo CPU and 2GB of memory. Because of the complexity of the model, only small instances are experimented.
The academic case study is illustrated in Figure 3. Two markets are considered: United States (US) and Europe (E). Production is possible in each country plus on a relocated unit in North Africa (NA), with labour rate of respectively 30 (US), 25(E) and 10 (NA)$\$_{/h}$. Distribution centers and customers are set randomly. One supplier per component type is set randomly close to each production unit.

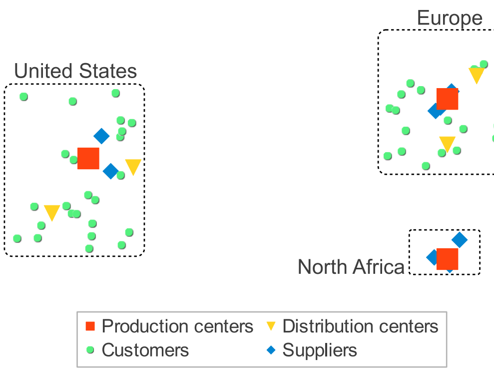

Figure 3: The supply chain of the case study.

Table 2 shows the parameters used in this case study. Part transportation cost is computed with the part volume, the distance between the units and the logistical cost. As the model considers a mono period, all fixed costs represents the use on this period. Product family is generated from the root product described in Figure 4: each final product is an instantiation of this root product with one or none alternative of each component. Three types of components are defined, each suppliers can supply all products of a given type. Substitutions are authorized when a part has more or better components than another.

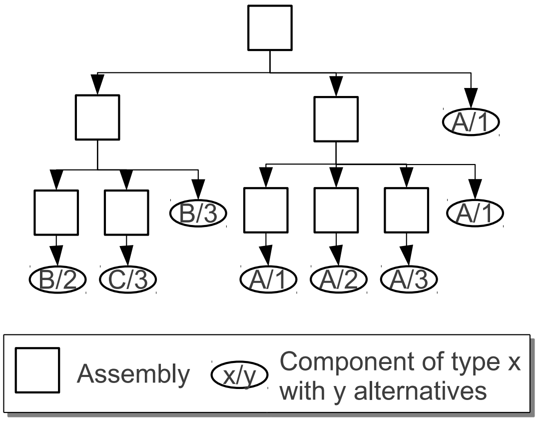

Figure 4: Root product from which the product family is generated.

Table 2: Case study characteristics.

Type	Parameter	Value
Network node	Logistical cost	150 $\$_{/m^3}$
Customers	Quantity	50
	Prob demand	.3
	Max demand per product	600
Fixed costs	per (axe, product)	100 $
	per (component, supplier)	500 $
	per (product, plant)	13,000 $
	per suppliers	10,000 $
	per plants	50,000 $
	per DC	20,000 $
Product parts	Final products	10
	Max procurement cost	20 $
	Max physical volume	1 m^3

Results are assessed through three criteria: percentage of labour allocation in each production center, extra costs compared to the optimal solution without constraint, and the commonality index of the manufactured family product. Many indexes have been proposed in the literature [11]. In this paper, the Extra Commonality Index (ECI) is proposed to measures the commonality brought by substitution decisions:

$$\text{ECI} = 1 - \frac{\#\text{ parts used} - \#\text{ parts min}}{\#\text{ parts max} - \#\text{ parts min}}$$

Parts contain components, high level assemblies and final products. The minimum number of parts refers to the solution with a unique final product, the maximum refers to the initial BOM. ECI is between 0 (no extra commonality) and 1 (total commonality).

3.2 Multi-sourcing

Multi-sourcing is a strategy that firms use to lower risk in procurement of key components. It consists in multiplying suppliers to avoid disruption if one supplier is deficient. At the opposite, a company can prefer to limit its suppliers. For each component $p \in \mathcal{R}$, the number of different suppliers that provide p is between b_{min}^p and b_{max}^p.

Constraint (14) defines decision variables M^p.

$$\sum_{i \in \mathcal{S}} B_i^p \leq M^p \qquad \forall p \in \mathcal{R} \tag{14}$$

Constraint (15) assures that the minimum and the maximum number of suppliers for each component is satisfied if the component is used.

$$M^p b_{min}^p \leq \sum_{i \in \mathcal{S}} B_i^p \leq M^p b_{max}^p \qquad \forall p \in \mathcal{R} \tag{15}$$

Constraint (16) avoid suppliers to provide less than Q_{min}^p units for part p.

$$B_i^p Q_{min}^p \leq A_i^p \qquad \forall p \in \mathcal{P}, \forall i \in \mathcal{S} \tag{16}$$

These constraints can also be expressed for production: a critical assembly can be forced to be produced in multiple production centers. In this paper, only constraints on suppliers are considered.
Experiments consist in considering different levels of sourcing: only one supplier per component, up to 2, exactly 2, up to 3, 2 or 3, and exactly 3. Constraints are even for each component. Results are presented in Figure 5.

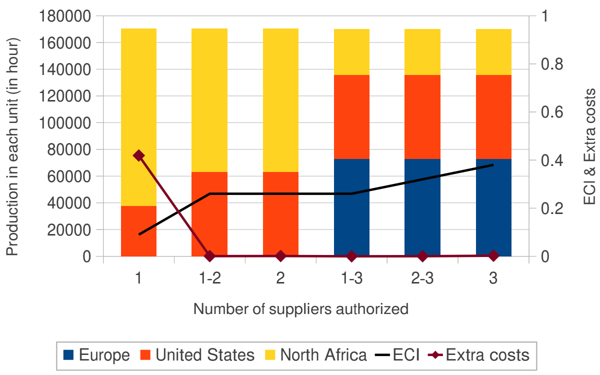

Figure 5: Allocation repartition, extra commonality index and extra costs according to multi-sourcing strategy.

Experiments shows that, when number of suppliers is constrained to be up to three (1-3 on Figure 5), there is not additional constraints on the problem as there can be one supplier per component type per unit, thus the result is the optimal. In this condition, some parts of the products (20%) are manufactured in the North-African unit while the rest of the production in done on the local units (43% in Europe, 37% in US). The allocation is the same when the number of suppliers is constrained to be higher: then all units must have a supplier for each type of product and substitutions are more important in both American and European units. On the contrary, when suppliers are constrained up to two, the European unit is unused and its production is absorbed by the North-African unit. When only one supplier is allowed, the North-African unit produces extra modules for the American unit and product standardization is very low. In this only case, the solution is significantly more expensive (40%).
A trend can be noted between standardization level and local production level. When production is concentrated far from the market, product standardization is low, whereas many substitution happen when local production units are important. This comes from economies of scale when production is concentrated in one unit.

3.3 Minimum activity levels

For some reasons, management can decide to keep a minimum activity level on some production center. This constraint can arise from legislative decisions, through subventions, or to avoid mass lay-offs that have major economical and image impacts that are hardly assessable. This can lead to continue production in center where it would be sub-optimal. In this case, which type of operation is kept ? When dealing with relocation, high-value added processes can be allocated to countries where labour rate is high but which are near the market, and high labour processes to cheaper countries.
Constraint (17) assures the desired minimum activity level on each production center.

$$\sum_{p \in \mathcal{M} \cup \mathcal{F}} l^p A_i^p \geq a_i \qquad \forall i \in \mathcal{U} \tag{17}$$

Experiments consist in forcing a minimum activity on the European unit, from 0 to the maximum quantity of work needed for the whole production. The case study is slightly different with transportation cost of 100 instead of 150 and demands of $2/3$ lower to have more pertinent results. Results are presented in Figure 6.

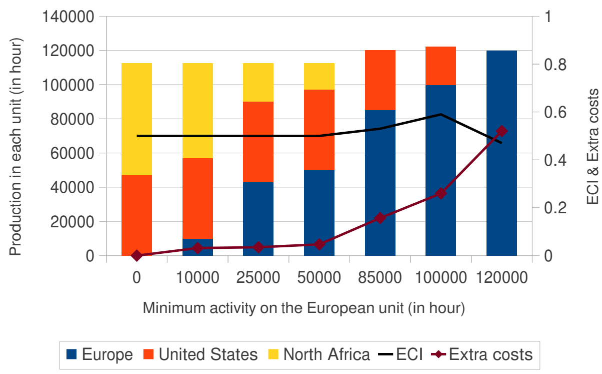

Figure 6: Allocation repartition, extra commonality index and extra costs according to minimum activity constraints on European unit.

Without any constraint, the European unit should close in the optimal solution. While constraining a minimum activity on this unit, two phenomena arise: till 50 000 hours, which represents the production for the European market, production is relocated from the North-African unit to the European one without a significant increase of the production costs. At 10 000 hours, the constraint is barely satisfied: only a little sub-assembly is produced then shipped to the American unit . At 25 000 hours, only one final product and its assemblies are manufactured in the North-African unit, the European unit produced all the others product to response its demand. Beyond 50 000 hours, the European unit have to produce for the American market, thus costs are increasing drastically, up to 152% of the optimal solution. The less production are kept in the American unit, the more substitutions increase in this unit while substitutions are stable in the European unit.

4 Conclusion

In this paper, we provide tools and a framework to analyze and quantify the influence of exogenous strategy on the mathematical optimization of the product and the supply chain. When dealing with qualitative issues, their economic impact assessment is relevant in the decision process and can rebuild all the decisions concerning the supply chain and the way the products are manufactured. A perspective is to internalize these strategies to become part of the decision process.

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