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# Improved fuzzy ranking procedure for decision making in product design

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In this paper, we present a method for ranking any number of normal fuzzy numbers using trapezoidal fuzzy numbers as a general form, where rectangular and triangular fuzzy numbers are particular cases of such a form. This general form is supported by 29 cases, which is enough to consider all the possible situations between two normal fuzzy numbers, such as trapezoidal, triangular, or rectangular. The ranking procedure is performed using four ordering criteria into a pseudo-order preference model considering the type of the fuzzy preference relation. Two examples are given to illustrate and validate the applicability and practicality of this fuzzy ranking method. A comparison and an analysis of the proposed method is presented to demonstrate its usefulness and its contribution to the improvement of the decision making processes as a result of its management of vague or imprecise information, and whether or not that information should be allowed to be entered into such processes.

**Keywords:** fuzzy ranking; fuzzy decision making; fuzzy numbers; fuzzy preference relations

## 1. Introduction

Manufacturing and service organisations are always making decisions. Although they are made at different levels: strategic, tactical, or operational, in the end, all are highly important to the successful achievement of organisational goals. The decision making process plays an important role in the success of both for-profit and not-for-profit companies, and for that it needs to be improved continuously. Because fuzzy logic is capable of managing imprecise information, and this capability is a critical aspect in the improvement of the decision making process, fuzzy logic has been increasingly used in decision making methods. The application of fuzzy logic to several decision making tools permits the consideration of imprecise information from the input variables which can be given in linguistic terms, such as 'very important', 'very high', 'medium', 'very low', and so on, aimed at representing variables from the human perspective.

Generally, the decision making process seeks to make decisions as a function of two or more variables, such as different characteristics, or alternatives, given in numerical and/or linguistic form. To consider this kind of variable, it is necessary to 'fuzzify' them by defining a fuzzy number for each, and is a process which should be performed by individuals with enough expertise to translate the linguistic information accurately.

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The ranking or ordering of this kind of number may seem to be a task that is easy to perform visually, but, in this work, we seek to replace human intervention in the ranking procedure with an appropriate alternative method.

This work is aimed at contributing to the fuzzy ranking procedure by simplifying the ordering process using the pseudo-order preference model and a set of ordering criteria. We also contribute by presenting a complete illustration of the method and list all the possible situations (29) that may occur between two normal trapezoidal fuzzy numbers which are capable of supporting any normal fuzzy number, such as a triangular or rectangular one. This paper is organised in the following sections. Section 2 presents the state of the art of the fuzzy decision making process and the importance of fuzzy logic in such a process. Section 3 presents the improved procedure for ranking fuzzy numbers and a simplification of the ordering procedure. Section 4 illustrates the application of the proposed ranking procedure. Section 5 presents a comparison and analysis of this and other methods. Section 6 concludes the paper.



## 2. Fuzzy logic and the decision making process

The decision making process plays an important role in the success of any company, and practically every engineering process involves different iterative and complex decision making activities. As a result, a great deal of research has been conducted on fuzzy decision making, including the use of fuzzy optimum selection (Dong *et al.* 2001), fuzzy multiple-attribute decision making (Kuo *et al.* 2006), and multiple-attribute decision making in concurrent engineering (Jiang and Chi-Hsing 2001).

Various approaches have been proposed to contribute to decision making. One of these is the fuzzy multicriteria decision making process (Fan *et al.* 2002, Büyüközkan and Feyzioğlu 2005, Işıklar and Büyüközkan 2006, Zhang *et al.* 2007), which is based principally on the fuzzy preference relation. Another is the approach proposed by Sun and Wu (2006) for the ranking process based on an easy and intuitive fuzzy simulation analysis method.

The basis for the decision making process is the ranking of fuzzy numbers (Lee and You 2003), and Baas and Kwakernaak (1977), and Chen and Klein (1994) proposed the application of fuzzy ranking for multicriteria decision making.

Several of the fuzzy ranking methods that have been developed include that of Chen and Klein (1994), who applied the $\alpha$-cut and fuzzy subtraction operations to calculate the area under the new fuzzy number, and that of Wang and Parkan (2005), who introduced three optimisation models to assess the relative importance weights of attributes in a multiple-attribute decision making problem.

The fuzzy preference relation has been widely used in fuzzy ranking (Delgado *et al.* 1988, Lee 2000, Modarres and Sadi-Nezhad 2001). Other works include the application of some specific concepts such as triangular membership functions (Chang 1981), and set maximisation and minimisation (Chen 1985). Lee and You (2003) presented a fuzzy ranking method for fuzzy numbers which considers a number of interesting functions and indices, such as the fuzzy satisfaction function, the fuzzy evaluation value, the degree of defuzzification, the degree of evaluation, and relative defuzzification indices. A novel method incorporating fuzzy preferences and range reduction techniques was proposed by Ma and Li (2008). Yuan (1991) presented four criteria for evaluating fuzzy ranking methods (fuzzy preference presentation, the rationality of preference ordering, distinguishability, and robustness), and suggested an improved ranking method based on the fuzzy preference relation.

Ranking methods based on the fuzzy preference relation have demonstrated their applicability in various areas. For example, Jiao and Tseng (1998) proposed a fuzzy ranking methodology for concept evaluation, which makes it possible to evaluate a conceptual design in the context of mass customisation; that is, given a set of alternatives, evaluate and select the alternative that can satisfy customer needs and design requirements considering the technical capabilities of the company. According to Tseng and Klein (1988, 1989), many ranking methods for fuzzy numbers have been developed. However, these methods fail to consider many important factors, such as shapes, ranking order, the relative preference or dominance of fuzzy numbers, and the ease of computation of the ranking algorithm. This has made it necessary to develop a new, accurate, effective, and efficient algorithm capable of ranking a large number of fuzzy numbers. More recently, Lee (2000) announced that the various methods for ranking fuzzy numbers could be classified into two categories. The first is based on defuzzification, and the second is based on the fuzzy preference relation. Lee (2000) also maintains that a good ranking method should satisfy the following four criteria: a fuzzy preference presentation, rationality of preference ordering, robustness, and efficiency.

Unfortunately, while interesting, these methods have some limitations, and currently there is no general model for the ranking process. This paper proposes to contribute to remedying this situation by proposing a procedure for ranking any number of normal fuzzy numbers, which extends the previous illustration and statement based on normal triangular fuzzy numbers (Tseng and Klein 1989). The proposed extensions use trapezoidal normal fuzzy numbers as a general base which supports both triangular and rectangular fuzzy numbers at the same time for ranking any number of fuzzy numbers.



## 3. Fuzzy ranking procedure

This section first describes the modelling of rectangular, triangular, and trapezoidal normal fuzzy numbers in the general model. Then, it describes how to use this modelling to rank any situation in which there are two fuzzy numbers, but also to rank any fuzzy numbers.

### 3.1 *Trapezoidal fuzzy numbers as a general form*

The improved ranking procedure in the fuzzy decision making process presented in this work is based on the algorithm proposed by Tseng and Klein (1989). In our work here, we extend this illustration by presenting a complete general form, using trapezoidal fuzzy numbers as a base, in which rectangular and triangular fuzzy numbers are particular cases of this general form.

Let A and B be two normal and convex trapezoidal fuzzy numbers where the support of A is the interval (a, d) and the support of B is the interval (e, h). The triangular fuzzy number is a particular case of the general form, when $b = c$ for fuzzy number A or $f = g$ for fuzzy number B. Rectangular fuzzy numbers (crisp interval) are possible when $a = b$ and $c = d$ for fuzzy number A, or when $e = f$ and $g = h$ for fuzzy number B. Also, constant values or crisp values are possible through the fuzzy line when $a = b = c = d$ and

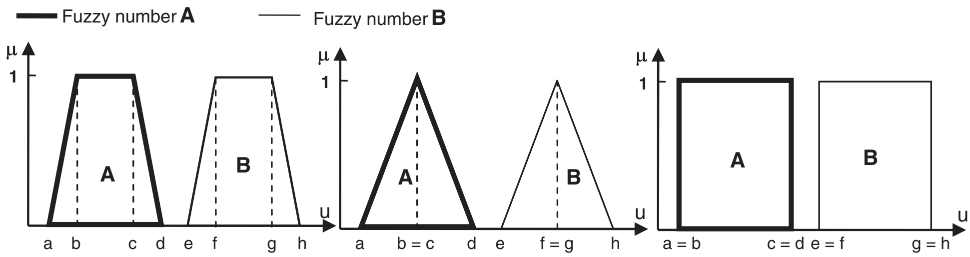


Figure 1. Triangular and rectangular fuzzy numbers as intrinsic cases of the trapezoidal form.

e = f = g = h for A and B, respectively. Figure 1 illustrates how trapezoidal fuzzy numbers can be used as a general model for triangular and rectangular fuzzy numbers.

The rankings of all possible situations of these fuzzy numbers (see Figure 2) are supported with the 29 cases depicted in Appendix 1, and the following section explains the ranking procedure for all of them.

Figure 2 shows that the extended illustration, based on trapezoidal fuzzy numbers, is able to rank any pairwise situation of trapezoidal, triangular, and rectangular fuzzy numbers.



### 3.2 *Definitions*

The proposed ranking procedure extends previous Tseng and Klein (1989) results. This extension is aimed at showing how it is possible to rank all possible pairwise situations of two normal fuzzy numbers (see Figure 2). To do this, some important concepts, such as indifference and dominance, overlap and non-overlap areas, and the fuzzy preference relation must be defined.

#### 3.2.1 *Definition 1*

If we let A and B be two normal and convex fuzzy numbers, then there exist the notions of indifference and dominance. These notions are defined as follows:

(1) If there exists an area of overlap between fuzzy numbers A and B (A and B intersect), then the overlap area is defined as indifference; that is to say, A and B are indifferent relative to one another in this area.
(2) If there exist one or more non-overlap areas between fuzzy numbers A and B, then the non-overlap areas represent the areas where either A dominates B or B dominates A.

The above notions for the general trapezoidal fuzzy numbers A and B can be identified in Figure 3. Cases (a) and (c) show the notion of dominance. This means that, for case (a), B dominates A, and for case (c), A dominates B. Case (b) represents the notion of indifference represented by the area of intersection between fuzzy numbers A and B. The domination between fuzzy numbers is given by the directions of A and B. In Appendix 1, cases (1) and (2) represent the non overlap situations and cases (3) to (29) represent the overlap situations for the general form.

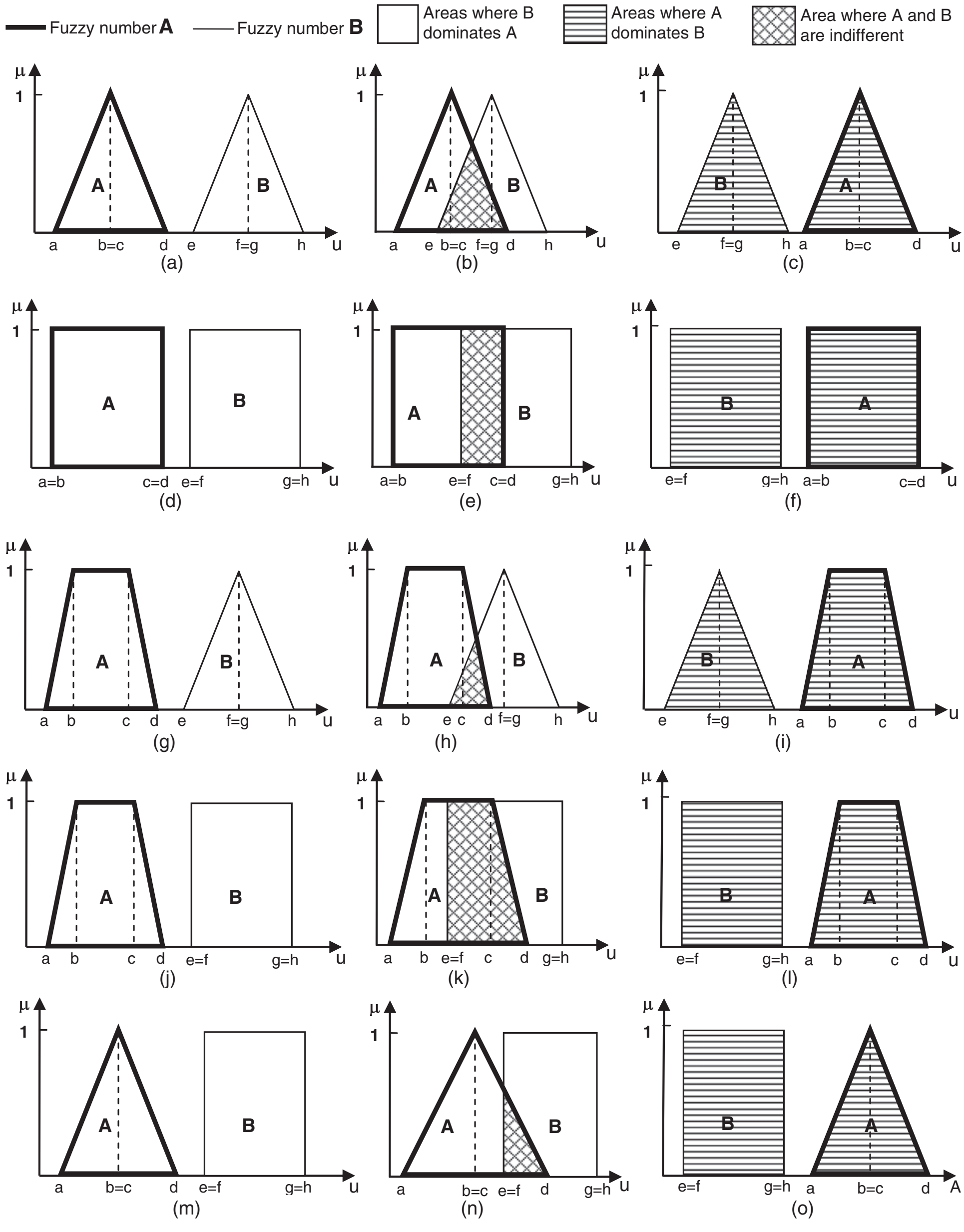


Figure 2. Possible pairwise situations of two normal fuzzy numbers.

### 3.2.2 *Definition 2*

If A and B are two fuzzy numbers, then R(A, B) and R(B, A) are two fuzzy preference relations and are defined as follows:

$$R(A, B) = \frac{(\text{areas where A dominates B}) + (\text{area where A and B are indifferent})}{(\text{area of A}) + (\text{area of B})}$$

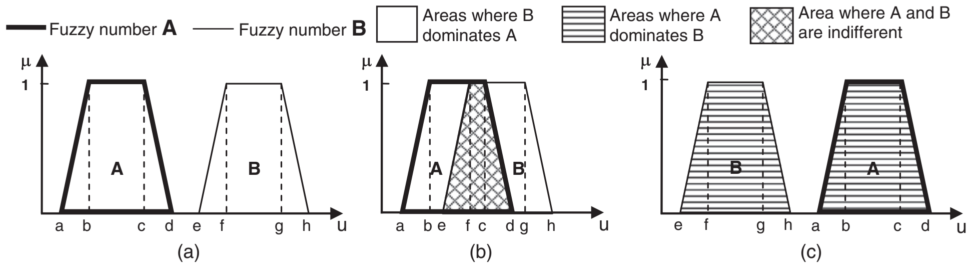


Figure 3. Depiction of notions. dominance and indifference between two fuzzy numbers.



$$R(B, A) = \frac{(\text{areas where B dominates A}) + (\text{area where A and B are indifferent})}{(\text{area of A}) + (\text{area of B})},$$

where R(A, B) and R(B, A) are interpreted as the degree to which A is preferred to or indifferent to B, and B is preferred to or indifferent to A respectively. R(A, B) and R(B, A) are reciprocal; that is to say, $R(A, B) + R(B, A) = 1$.

Based on the definitions of dominance and indifference, the following algorithm can be used to determine a preference relation (Tseng and Klein 1989).

**Algorithm**

**Step 1:** Find the area where A and B intersect.

**Step 2:** Find the areas where A dominates B.

**Step 3:** Find the areas where B dominates A.

**Step 4:** Find the areas of A and B.

**Step 5:** Compute the fuzzy preference relations R(A, B) and R(B, A).

#### 3.2.3 *Definition 3*

From Definition 2, the non-overlap areas between two fuzzy numbers must be obtained. In our work here, we use the Hamming distance for this purpose, which makes it possible to determine the areas where A dominates B and the areas where B dominates A, as needed in Steps 2 and 3 of the above algorithm. Here, we illustrate this concept considering normal trapezoidal, triangular, and rectangular fuzzy numbers. To determine the intervals of dominance on the real line for the two fuzzy numbers A and B, four cases must be considered. These cases depend on the number of intersections between the fuzzy numbers. There are five possibilities: four, three, two, one, and zero point(s) of intersection. Let the intersection points of the fuzzy numbers A and B be given by $X_1$, $X_2$, $X_3$, and $X_4$, where $X_1 < X_2 < X_3 < X_4$ (see Appendix 1). Tables 1 to 5 show how the Hamming distance can be obtained for each general case.

From Tables 1 to 5, 29 general cases are defined, based on Appendix 1. These cases are capable of supporting any pairwise situation of normal trapezoidal, triangular, or rectangular fuzzy numbers. Table 1 illustrates the four cases where it is possible to have four points of intersection between two trapezoidal fuzzy numbers. Table 2 shows the 13 cases where it is possible to have three points of intersection. Table 3 presents the eight

Table 1. Cases where it is possible to have four points of intersection.

| Case | Four intersection points are only possible if either: | The Hamming distance $D(A,B)$ for each case can be obtained as follows: |
|---|---|---|
| 10 | $a \ge e,\ b<f,\ c<g,\ d \ge h,\ c>f$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_4}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 13 | $a \ge e,\ b<f,\ c>g,\ d \le h$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{x_4}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 23 | $a \le e,\ b>f,\ c>g,\ d \le h,\ b<g$ | $D(A,B)=\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{x_4}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 28 | $a \le e,\ f \le b,\ c \le g,\ h \le d$ | $D(A,B)=\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_4}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |

Table 2. Cases where it is possible to have three points of intersection.

| Case | Three intersection points are only possible if either: | The Hamming distance $D(A,B)$ for each case can be obtained as follows: |
|---|---|---|
| 6 | $a \ge e,\ b<f,\ c<g,\ d \ge h,\ c<f$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 8 | $a \ge e,\ b<f,\ c<g,\ d \le h,\ c>f$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 9 | $a \le e,\ b<f,\ c<g,\ d \ge h,\ c>f$ | $D(A,B)=\int_{x_3}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 12 | $a \ge e,\ b<f,\ c>g,\ d \ge h,\ b<g$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 14 | $a \le e,\ b<f,\ c>g,\ d \ge h,\ c>f$ | $D(A,B)=\int_{x_2}^{x_3}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 15 | $a \ge e,\ b<f,\ c=g,\ d \ge h,\ b<g$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 16 | $a \ge e,\ b<f,\ c=g,\ d \le h,\ b<g$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 18 | $e \le a,\ f=b,\ g<c,\ d \le h,\ b<g$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_2}^{x_3}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 20 | $a \le e,\ b=f,\ c>g,\ d \le h,\ c>f$ | $D(A,B)=\int_{x_2}^{x_3}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 21 | $a \le e,\ b>f,\ c>g,\ d \le h,\ b>g$ | $D(A,B)=\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_2}^{x_3}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 22 | $a \le e,\ b>f,\ c>g,\ d \ge h,\ b<g$ | $D(A,B)=\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 27 | $a \le e,\ f \le b,\ c \le g,\ d \le h$ | $D(A,B)=\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 29 | $e \le a,\ f \le b,\ c \le g,\ h \le d$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_3}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |

Table 3. Cases where it is possible to have two points of intersection.

| Case | Two intersection points are only possible if either: | The Hamming distance $D(A,B)$ for each case can be obtained as follows: |
|---|---|---|
| 4 | $a \ge e,\ b<f,\ c<g,\ d \le h,\ c<f$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 5 | $a \le e,\ b<f,\ c<g,\ d \ge h,\ c<f$ | $D(A,B)=\int_{x_2}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| | | $D(A,B)=\int_{x_2}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 7 | $a \le e,\ b \le f,\ c \le g,\ d \le h,\ f \le c$ | $D(A,B)=0$ |
| 11 | $a \le e,\ b \le f,\ g \le c,\ h \le d$ | $D(A,B)=\int_{x_2}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 17 | $e \le a,\ f=b,\ g=c,\ h \le d$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_2}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 19 | $a \ge e,\ b=f,\ c=g,\ d \le h,\ b<g$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 25 | $a \le e,\ b>f,\ c>g,\ d \ge h,\ b>g$ | $D(A,B)=\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_2}^{d}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |
| 26 | $a \ge e,\ b>f,\ c>g,\ d \le h,\ b>g$ | $D(A,B)=\int_e^{x_1}\lvert\mu_A(u)-\mu_B(u)\rvert du+\int_{x_1}^{x_2}\lvert\mu_A(u)-\mu_B(u)\rvert du$ |

Table 4. Cases where it is possible to have one point of intersection.

| Case | One intersection point is only possible if either: | The Hamming distance $D(A, B)$ for each case can be obtained as follows: |
|---|---|---|
| 3 | $a \leq e,\ b < f,\ c < g,\ d \geq e,\ c < f$ | $D(A, B) = 0$ |
| 24 | $a \geq e,\ b > f,\ c > g,\ d \geq h,$ $a \leq h,\ b > g$ | $D(A, B) = \int_e^{x_1} \lvert\mu_A(u) - \mu_B(u)\rvert \mathrm{d}u + \int_{x_1}^{d} \lvert\mu_A(u) - \mu_B(u)\rvert \mathrm{d}u$ |

Table 5. Cases where it is not possible to have any point of intersection.

| Case | No intersection point is possible only if either: | The Hamming distance $D(A, B)$ for each case can be obtained as follows: |
|---|---|---|
| 1 | $a < e,\ b < f,\ c < g,\ d \leq e$ | $D(A, B) = 0$ |
| 2 | $a > e,\ b > f,\ c > g,\ a \geq h$ | $D(A, B) = \int_e^{h} \lvert\mu_A(u) - \mu_B(u)\rvert \mathrm{d}u + \int_{a}^{d} \lvert\mu_A(u) - \mu_B(u)\rvert \mathrm{d}u$ |



cases where it is possible to have two points of intersection. Table 4 shows the two cases where it is possible to have one point of intersection. Finally, Table 5 presents the two cases where it is not possible to have any point of intersection. All the cases presented in Tables 1 to 5 are depicted in Appendix 1.

### 3.2.4 *Definition 4*

Here, we apply a pseudo-order preference model for the fuzzy ranking procedure for two alternatives. This model has already been used in the literature several times (Roy and Vincke 1984, Wang 1997, Gungor and Arikan 2000, Büyüközkan and Feyzioğlu 2004). Let the fuzzy preference relation between two ideas $a$ and $b$ for criterion $i$ be obtained by the pairwise comparison of $g_i(a)$ and $g_i(b)$, which shows the linguistic performance of ideas $a$ and $b$ respectively. $g_i(a)$ and $g_i(b)$ are represented by fuzzy numbers. Three types of preference relation are defined in terms of the fuzzy preference relations between two alternatives $\forall a, b \in A$ and $i \in C$, as follows:

$$aP_ib \Leftrightarrow P(g_i(a), g_i(b)) - P(g_i(b), g_i(a)) > p_i,$$
$$aQ_ib \Leftrightarrow P(g_i(a), g_i(b)) - P(g_i(b), g_i(a)) \leq p_i,$$
$$aI_ib \Leftrightarrow \lvert P(g_i(a), g_i(b)) - P(g_i(b), g_i(a))\rvert \leq q_i,$$

where $P_i$ and $Q_i$ depict strict and weak preference, respectively, and $I_i$ depicts indifference. The preference threshold $p_i$ and the indifference threshold $q_i$ (defined by common sense, Roy and Vincke 1984) are used to discriminate between the indifference, strict preference, and weak preference of two alternatives for criterion $i$. The three possible types of preference should be read as follows:

- $aP_ib$, where there is a strict preference between ideas $a$ and $b$ (idea $a$ is strictly preferred to idea $b$ for criterion $i$).
- $aQ_ib$, where there is a weak preference between ideas $a$ and $b$ (idea $a$ is weakly preferred to idea $b$ for criterion $i$).
- $aI_ib$, where there is no difference between ideas $a$ and $b$ (idea $a$ is no different from idea $b$ for criterion $i$).

3.2.5 *Definition 5*

To extend Definition 4 for ranking more than two fuzzy numbers, the following four criteria procedure must be considered:

(1) Criterion 1. The largest number of strict preferences. The tie-breaker for this criterion is Criterion 2.
(2) Criterion 2. The largest number of weak preferences. The tie-breaker for this criterion is Criterion 3.
(3) Criterion 3. The smallest number of indifference situations. The tie-breaker for this criterion is Criterion 4.
(4) Criterion 4. If the fuzzy preference belongs to the indifference situation, then there is no difference between these fuzzy numbers, and these can be ranked indifferently.

To apply these criteria, some priority rules must be followed. Criterion 1 has priority one, and it must be applied as long as possible until there is a conflict and a tie-breaker becomes necessary. Criteria 2 and 3 have priority two and three respectively, and these should be applied in the same way as Criterion 1. Criterion 4 should be applied when the preference situation is a pairwise situation with indifference. Below we illustrate this procedure with an example.

## 4. Illustrative examples

### 4.1 *Ranking of any pairwise situation of fuzzy numbers*

The following example shows how the general form makes it possible to rank any pairwise situation (referred to hereafter as a 'pairwise') of two normal fuzzy numbers. Let us consider the following fuzzy numbers:

$A_1$: a trapezoidal fuzzy number $[1.5, 3, 4, 6]$;
$A_2$: a triangular fuzzy number $[2, 5, 5, 7]$;
$A_3$: a rectangular fuzzy number $[1.5, 1.5, 6, 6]$;
$B_1$: a trapezoidal fuzzy number $[5, 7, 8, 9]$;
$B_2$: a triangular fuzzy number $[2, 6, 6, 8]$;
$B_3$: a rectangular fuzzy number $[5, 5, 9, 9]$.

By considering the preference threshold $p_i = 0.85$ and the indifference threshold $q_i = 0.25$, as used in Wang (1997), the ranking of $A_1$ with $B_1$, $B_2$, and $B_3$ is given in the following subsections.

4.1.1 *Fuzzy ranking of $A_1$ and $B_1$*

The pairwise $A_1$–$B_1$ belongs to case 3, as depicted in Appendix 1 and defined in Table 4. This general case deals with the fuzzy preference relation between two trapezoidal fuzzy numbers, as shown in Figure 4.

Based on Definitions 1 to 3 in Section 3, the fuzzy preference relation for $A_1$–$B_1$ can be obtained by applying the following notation:

$\mathrm{Dom}(A_1, B_1)$: the area where $A_1$ dominates $B_1$;
$\mathrm{Dom}(B_1, A_1)$: the area where $B_1$ dominates $A_1$;
$\mathrm{Ind}(A_1–B_1)$: the area where $A_1$ and $B_1$ are indifferent;

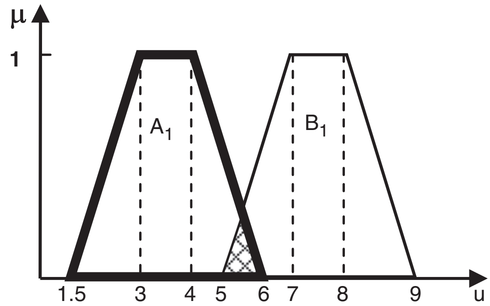


Figure 4. Pairwise of $A_1$–$B_1$.

Table 6. Fuzzy preference relations between two normal fuzzy numbers.

| Fuzzy number | Trapezoidal $A_1$ [1.5 3 4 6] | Triangular $A_2$ [2 5 5 7] | Rectangular $A_3$ [1.5 1.5 6 6] |
|---|---|---|---|
| Trapezoidal $B_1$ [5 7 8 9] | | | |
| R(A, B) | 0.0238 | 0.1 | 0.0357 |
| R(B, A) | 0.9762 | 0.9 | 0.9643 |
| Preference | $B_1 P A_1$ | $B_1 Q A_2$ | $B_1 P A_3$ |
| Triangular $B_2$ [2 6 6 8] | | | |
| R(A, B) | 0.2319 | 0.3788 | 0.2667 |
| R(B, A) | 0.7681 | 0.6212 | 0.7333 |
| Preference | $B_2 Q A_1$ | $B_2 I A_2$ | $B_2 Q A_3$ |
| Rectangular $B_3$ [5 5 9 9] | | | |
| R(A, B) | 0.037 | 0.1538 | 0.1176 |
| R(B, A) | 0.963 | 0.8462 | 0.8824 |
| Preference | $B_3 P A_1$ | $B_3 Q A_2$ | $B_3 Q A_3$ |

$\text{Area}(A_1)$: the area of $A_1$;
$\text{Area}(B_1)$: the area of $B_1$;
$X_i$: the points of intersection i.

Then,

$$R(A_1, B_1) = \frac{\text{Dom}(A_1, B_1) + \text{Ind}(A_1 - B_1)}{\text{Area}(A_1) + \text{Area}(B_1)}$$

$$R(B_1, A_1) = \frac{\text{Dom}(B_1, A_1) + \text{Ind}(A_1 - B_1)}{\text{Area}(A_1) + \text{Area}(B_1)}.$$

As $\text{Dom}(A_1, B_2) = 0$, $\text{Ind}(A_1–B_1) = 0.125$, $\text{Area}(A_1) = 2.75$, $\text{Area}(B_1) = 2.5$, and $X_1 = 5.5$, then, $R(A_1, B_1) = 0.0238$ and $R(B_1, A_1) = 0.9762$. Since $R(B_1, A_1) = 0.9762 > q_i$, then $B_1$ is strictly preferred to $A_1$ and is denoted $B_1 P A_1$ (see Table 6).

### 4.1.2 *Fuzzy ranking of $A_1$ and $B_2$*

The pairwise $A_1$–$B_2$ also belongs to case 3. This general case deals with the fuzzy preference relation between two trapezoidal fuzzy numbers, as shown in Figure 5.

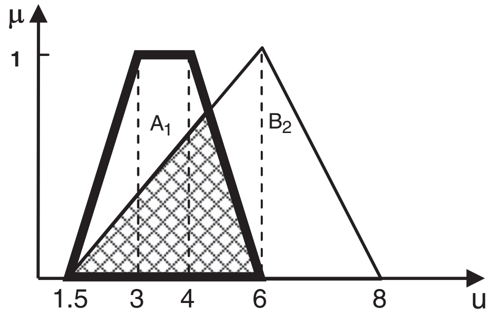


Figure 5. Pairwise of $A_1$–$B_2$.

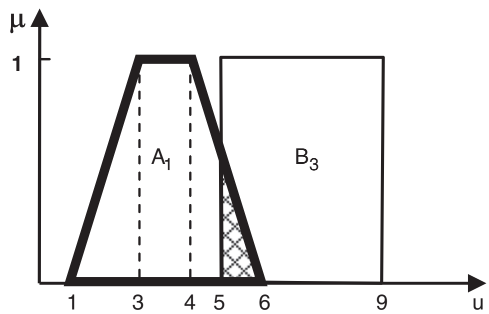


Figure 6. Pairwise of $A_1$–$B_3$.



As $Dom(A_1, B_2) = 0$, $Ind(A_1–B_2) = 1.333$, $Area(A_1) = 2.75$, $Area(B_2) = 3$, and $X_1 = 4.6667$, then, $R(A_1, B_2) = 0.2319$ and $R(B_2, A_1) = 0.7681$. Since $R(B_2, A_1) = 0.7681 \leq q_i$, then $B_2$ is weakly preferred to $A_1$ and is denoted $B_2QA_1$ (see Table 6).

#### 4.1.3 *Fuzzy ranking of $A_1$ and $B_3$*

The pairwise $A_1$–$B_3$ also belongs to case 3. This general case deals with the fuzzy preference relation between two trapezoidal fuzzy numbers, as shown in Figure 6.

As $Dom(A_1, B_3) = 0$, $Ind(A_1–B_3) = 0.25$, $Area(A_1) = 2.75$, $Area(B_3) = 4$, and $X_1 = 5$, then, $R(A_1, B_3) = 0.0370$ and $R(B_3, A_1) = 0.9630$. Since $R(B_3, A_1) = 0.9630 > q_i$, then $B_3$ is strictly preferred to $A_1$ and is denoted $B_3PA_1$ (see Table 6).

#### 4.1.4 *Synthesis*

Table 6 summarises the values and types of fuzzy preference relations for some possible situations arising between predefined fuzzy numbers.

Table 6 shows the three types of fuzzy preference relation defined for the pseudo-order preference model in Section 3. These types are:

(1) The strict preference relation belonging to the pairwise(s) $A_1$–$B_1$, $A_1$–$B_3$, and $A_3$–$B_1$;

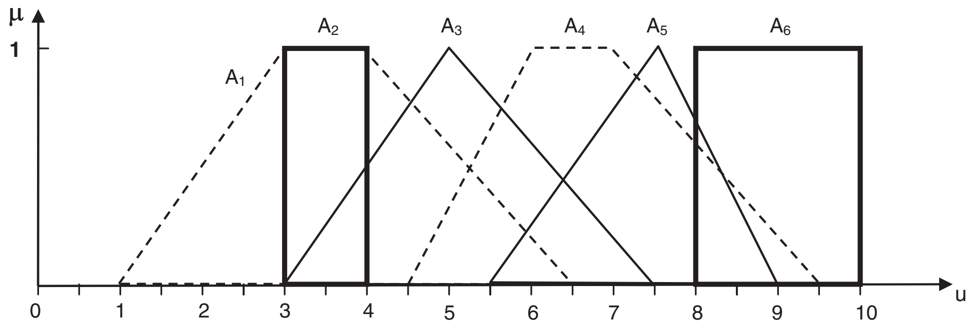


Figure 7. Ranking of multiple fuzzy numbers.



(2) The weak preference relation belonging to the pairwise(s) $A_1$–$B_2$, $A_2$–$B_1$, $A_2$–$B_3$, $A_3$–$B_2$, and $A_3$–$B_3$;
(3) The indifference situation, which belongs to the pairwise $A_2$–$B_2$.

This example shows the advantage of using the general form proposed in this paper to rank any pairwise of normal fuzzy numbers, whether they are trapezoidal, triangular, or rectangular. This example also shows how the use of the pseudo-order preference model improves and simplifies the ranking procedure used by Tseng and Klein (1989).

### 4.2 *Ranking more than two fuzzy numbers*

The ranking of more than two fuzzy numbers can be achieved by applying the four criteria procedure presented in Definition 5 in Section 3. Let us consider the six normal fuzzy numbers depicted in Figure 7 as different alternatives: $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, and $A_6$, in a decision making process:

$A_1$: a trapezoidal fuzzy number $[1, 3, 4, 6.5]$;
$A_2$: a rectangular fuzzy number $[3, 3, 4, 4]$;
$A_3$: a triangular fuzzy number $[3, 5, 5, 7.5]$;
$A_4$: a trapezoidal fuzzy number $[4.5, 6, 7, 9.5]$;
$A_5$: a triangular fuzzy number $[5.5, 7.5, 7.5, 9]$;
$A_6$: a rectangular fuzzy number $[8, 8, 10, 10]$.

To apply these criteria, the fuzzy preference relation for all the possible pairwises between them must be obtained. This information is presented in Table 7.

Also, the type of fuzzy preference must be considered for each pairwise. Table 8 presents the type of fuzzy preference relation for each pairwise among all the alternatives (the same preference threshold $p_i = 0.85$ and indifference threshold $q_i = 0.25$ are used).

To apply the proposed procedure, the frequency and type of fuzzy preference relation must be obtained for each pairwise of alternatives. This information is shown in Table 9.

Let us apply the proposed multi-ranking procedure:

(1) Criterion 1. The largest number of strict preferences: by applying Criterion 1 from the proposed procedure, the largest number of strict preferences belongs first to $A_6$, then to $A_5$, and finally to $A_4$. Now consider that $(A > B)$ expresses that A is

Table 7. Fuzzy preference relation.

| Fuzzy number | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | 0.5 | 0.5294 | 0.2475 | 0.08 | 0 | 0 |
| $A_2$ | 0.4706 | 0.5 | 0.0769 | 0 | 0 | 0 |
| $A_3$ | 0.7525 | 0.9231 | 0.5 | 0.2143 | 0.1111 | 0 |
| $A_4$ | 0.92 | 1 | 0.7857 | 0.5 | 0.3743 | 0.09 |
| $A_5$ | 1 | 1 | 0.8889 | 0.6257 | 0.5 | 0.0889 |
| $A_6$ | 1 | 1 | 1 | 0.91 | 0.9111 | 0.5 |

Table 8. Type of fuzzy preference relation.

| Fuzzy number | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | – | $A_2IA_1$ | $A_3QA_1$ | $A_4QA_1$ | $A_5PA_1$ | $A_6PA_1$ |
| $A_2$ | $A_2IA_1$ | – | $A_3QA_2$ | $A_4PA_2$ | $A_5PA_2$ | $A_6PA_2$ |
| $A_3$ | $A_3QA_1$ | $A_3QA_2$ | – | $A_4QA_3$ | $A_5QA_3$ | $A_6PA_3$ |
| $A_4$ | $A_4QA_1$ | $A_4PA_2$ | $A_4QA_3$ | – | $A_5QA_4$ | $A_6QA_4$ |
| $A_5$ | $A_5PA_1$ | $A_5PA_2$ | $A_5QA_3$ | $A_5QA_4$ | – | $A_6QA_5$ |
| $A_6$ | $A_6PA_1$ | $A_6PA_2$ | $A_6PA_3$ | $A_6QA_4$ | $A_6QA_5$ | – |



Table 9. Frequency and type of fuzzy preference relation for each alternative.

| | Frequency | | |
|---|---|---|---|
| Alternative | *P* | *Q* | *I* |
| $A_1$ | 0 | 0 | 1 |
| $A_2$ | 0 | 0 | 1 |
| $A_3$ | 0 | 2 | 0 |
| $A_4$ | 1 | 2 | 0 |
| $A_5$ | 2 | 2 | 0 |
| $A_6$ | 3 | 2 | 0 |

preferred to B. The first part of the multiple fuzzy ranking is: $A_6 > A_5 > A_4$ (tiebreaker not needed).

(2) Criterion 2. The largest number of weak preferences: by applying Criterion 2 to the rest of the numbers, the largest number of weak preferences belongs to $A_3$, and so the new ranking is: $A_6 > A_5 > A_4 > A_3$ (tiebreaker not needed).

(3) Criterion 3. The smallest number of indifference situations: by applying Criterion 3 to the rest of the numbers, the smallest number of indifference situations belongs to $A_1$ and $A_2$. So, a tiebreaker is necessary. The tiebreaker for this criterion consists of the application of Criterion 4.

(4) Criterion 4. If the fuzzy preference belongs to the indifference situation, then there is no difference between the numbers. Finally, by applying Criterion 4 to the rest of the numbers, if the fuzzy preference belongs to the indifference situation, then there is no difference between $A_1$ and $A_2$. Hence, the final fuzzy ranking can be as

follows: $A_6 > A_5 > A_4 > A_3 > A_2 > A_1$ or $A_6 > A_5 > A_4 > A_3 > A_1 > A_2$, since $A_1$ and $A_2$ are indifferent.

This example shows how the pseudo-order preference model can be easily used to rank more than two fuzzy numbers by applying the four criteria proposed procedure to a set of different alternatives.

## 5. Analysis and comparison

The ranking of fuzzy numbers has been a concern in many different areas, principally those related to decision making processes. Over recent decades, several fuzzy ranking methods and approaches have been proposed aimed at contributing to the improvement of the decision making process.

According to Lee and Chen (2008), various classifications of ranking methods have been published, which are listed below.

- Tseng and Klein (1989), who classified the ranking methods based on:
  - Hamming distance;
  - Fuzzy boundaries;
  - Centroid index;
  - Possibility dominance;
  - Probability proportions.
- Chen and Hwang (1992), who grouped the methods into four major classes:
  - Preference relation;
  - Fuzzy mean and spread;
  - Fuzzy scoring;
  - Linguistic expression.
- In the same context, Lee (2000) argued that ranking methods can be classified into two principal categories:
  - Methods based on fuzzy preference relations;
  - Methods based on defuzzification techniques.

Here, a comparison of fuzzy ranking methods is presented. This comparison is based primarily on Lee's (2000) classification (see Table 10).

Table 10 presents a comparison of different fuzzy ranking methods considering 10 criteria grouped into four principal categories: the type of mathematical tool (MT), the type of membership function (MF), the size of fuzzy number set (FNS), and the type of fuzzy number. The 10 criteria considered are listed below.

(1) Fuzzy preference relation.
(2) Defuzzification.
(3) Other mathematical tools, such as: fuzzy simulation analysis (Sun and Wu 2006) and the range reduction technique with rank minimisation (Ma and Li 2008).
(4) Triangular membership function.
(5) Trapezoidal membership function.

Table 10. Comparison of fuzzy ranking methods.

| | Criteria | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Type of MT | | | Type of MF | | | Size of FNS | | Type of FN | |
| Method | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Tseng and Klein 1989 | * | | | * | | | * | * | * | |
| Yuan 1991 | * | | | * | | * | * | * | * | |
| Lee 2000 | * | | | * | | * | * | | * | |
| Modarres and Sadi-Nezhad 2001 | * | | | * | | | * | | * | |
| Smith and Verma 2004 | | * | | * | | | * | | * | |
| Sun and Wu 2006 | | | * | * | * | | * | | * | |
| Chen and Chen 2007 | | * | | * | * | | * | * | * | * |
| Ma and Li 2008 | | | * | * | | | * | | * | |
| Lee and Chen 2008 | | * | | * | * | * | * | * | * | * |
| Chen and Wang 2009 | | * | | * | * | * | * | * | * | * |
| Wu and Mendel 2009 | | * | | * | * | | | | * | * |
| Proposed method | * | | | * | * | * | * | * | * | |

(6) Other membership functions, such as: the parabolic membership function (Yuan 1991, Lee 2000) and the rectangular membership function or crisp interval (Chen and Wang 2009).
(7) Set of two fuzzy numbers.
(8) Set of more than two fuzzy numbers.
(9) Normal fuzzy numbers.
(10) Non-normal fuzzy numbers.

From Table 10, the fuzzy preference relation and defuzzification are two of the principal criteria (columns 1 and 2) that embrace different approaches and mathematical tools which make it possible to obtain the information needed to make decisions about how to order or rank variables considered as fuzzy numbers in this work. In column 3, some interesting ranking methods are considered that are not necessarily based on the fuzzy preference relation or on defuzzification. Other mathematical tools are applied in these methods, such as fuzzy simulation analysis (Sun and Wu 2006) and the range reduction technique with rank minimisation (Ma and Li 2008).

Owing to their practicality and adaptability, the triangular and trapezoidal membership functions (columns 4 and 5) are two of the most widely used for representing linguistic information as fuzzy numbers. Most of the published fuzzy ranking methods consider one or both of these. Other membership functions (column 6) have been considered to represent fuzzy numbers. These include the parabolic membership function (Lee 2000), the rectangular membership function, and the crisp interval (Chen and Wang 2009).

Frequently, the decision making process involves the evaluation of several variables. For that, it is highly important to have the capability to manage multiple variables in the ranking methods. Column 8 shows which methods are capable of considering multiple variables instead of just pairs (column 7) of them.

As can be noted in Table 10 (columns 9–10), the proposed method in this paper is limited to normal fuzzy numbers. This limitation is also presented in all the ranking methods based on the fuzzy preference relation. Also, it is possible to note that, for all the methods based on defuzzification techniques, this limitation is not presented. But, according to Lee (2000), ranking methods based on defuzzification satisfy only one of the four criteria that a good ranking method should satisfy. These four criteria are: (1) fuzzy preference representation; (2) the rationality of preference ordering; (3) robustness; and (4) efficiency. Methods based on the preference relation satisfy criteria (1), (2), and (3), whereas methods based on defuzzification only satisfy criterion (4).

The efficiency of a ranking method is an important aspect which should be considered at the time of choosing a ranking method, but at the same time aspects such as fuzzy preference representation, rationality, and robustness are equally important. Nowadays, as a result of the development of quicker and more accessible computation methods, efficiency can be less important than robustness and accuracy.

Fuzzy ranking methods have been widely applied in a number of areas, principally in multiple-attribute decision making processes. Jiao and Tseng (1998) applied the fuzzy preference relation to a fuzzy ranking methodology for conceptual design evaluation. Liqing *et al.* (2008) applied a fuzzy ranking approach, also based on the fuzzy preference relation, to consider customer preferences and technical capabilities in the evaluation of design schemes. More recently, Ho *et al.* (2009) applied fuzzy ranking based on the fuzzy preference relation to compare and prioritise a train services provider's bid to produce a negotiation sequence. These three ranking methods use the Hamming distance to obtain the relations. According to Ho *et al.* (2009) the Hamming distance approach is suitable because a shorter computation time is required.

As mentioned previously, our proposed method also uses the Hamming distance approach to obtain the fuzzy preference relation, as proposed by Tseng and Klein (1989). For this reason, we consider it convenient here to note some important aspects of the two methods.

This work proposes an algorithm to rank any number of normal and convex fuzzy numbers. Tseng and Klein (1989) argue that their method is capable of managing non-normal and non-convex fuzzy numbers, however this is a contradiction, because their method is defined considering two normal and convex fuzzy numbers. They justify the ability to manage non-normal and non-convex fuzzy numbers through the application of human comparison as part of the algorithm in steps 2 and 3.

It is difficult to include human or manual comparison in an autonomous program or intelligent system. For this reason, human comparison is not considered pertinent for our method.

The Tseng and Klein (1989) algorithm was tested on a set of 13 cases of paired examples. For most of them, the fuzzy numbers are normal and convex. But, for the last two examples (L and M), some of the fuzzy numbers do not satisfy these characteristics. In example L, one of the fuzzy numbers is not normal, and, in example M, one is not convex. The evaluation of these numbers was made possible through the application of human comparison in the algorithm.

Our method avoids manual (human) comparison by visually controlling the ranking procedure to permit proper application of the method in autonomous systems.

For normal and convex fuzzy numbers, the two methods are similar. The key here, though, is that our method is much easier to translate into a system because of the extended illustration of its general form.

The application of the pseudo-order preference model and the consideration of the preference type makes our method more accurate and easier to apply than that proposed by Tseng and Klein (1989) when ranking more than two fuzzy numbers.

In summary, our proposed method offers some interesting advantages over other proposed fuzzy ranking methods based on the fuzzy preference relation. The illustration and the mention of the 29 cases can be considered to constitute a framework for the development of new decision making systems based on the application of fuzzy logic. Crisp values and crisp intervals must sometimes be represented through fuzzy numbers, because they can represent some variables such as schedule time, speed, and so on. The proposed method is able to manage this situation through the fuzzy line when $a = b = c = d$, or the triangular fuzzy numbers when $a = b$ and $c = d$.

The depiction of the entire possible situation between two fuzzy numbers can be used as a framework for the development of statements to include other membership functions, such as Gaussian and parabolic, among others, and to include non-normal fuzzy numbers as well.



## 6. Conclusions

In this paper, an improved fuzzy ranking method has been presented, which could be used to make important decisions around different processes such as product design. The calculation of the fuzzy preference relation and the application of the pseudo-order preference model constitute the basis for this proposition. The type of fuzzy preference relation is used to rank more than two alternatives in an easy and practical way by applying a four criteria procedure. Two illustrative examples are presented, the first to show the capability of the improved procedure to rank rectangular, triangular, and trapezoidal fuzzy numbers, and the second to demonstrate the application of the proposed procedure to rank more than two normal fuzzy numbers in a practical way by exploiting the type of preference through the application of the ordering criteria. The comparison and analysis of the proposed method and others makes it possible to demonstrate the usefulness of our proposal. The application of fuzzy logic to the proposed method makes it possible for decision makers to profit from information expressed in linguistic terms which are frequently vague and imprecise.

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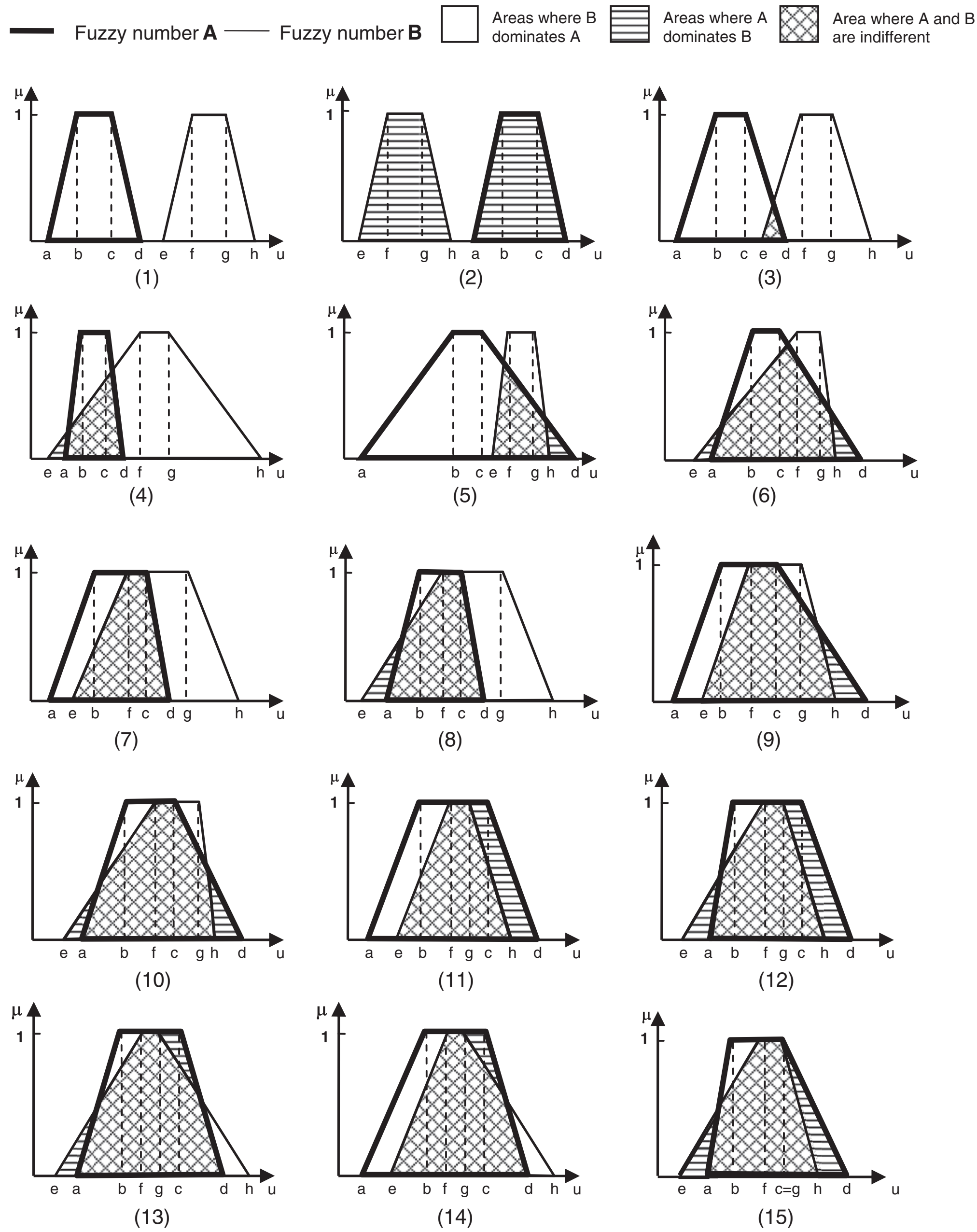
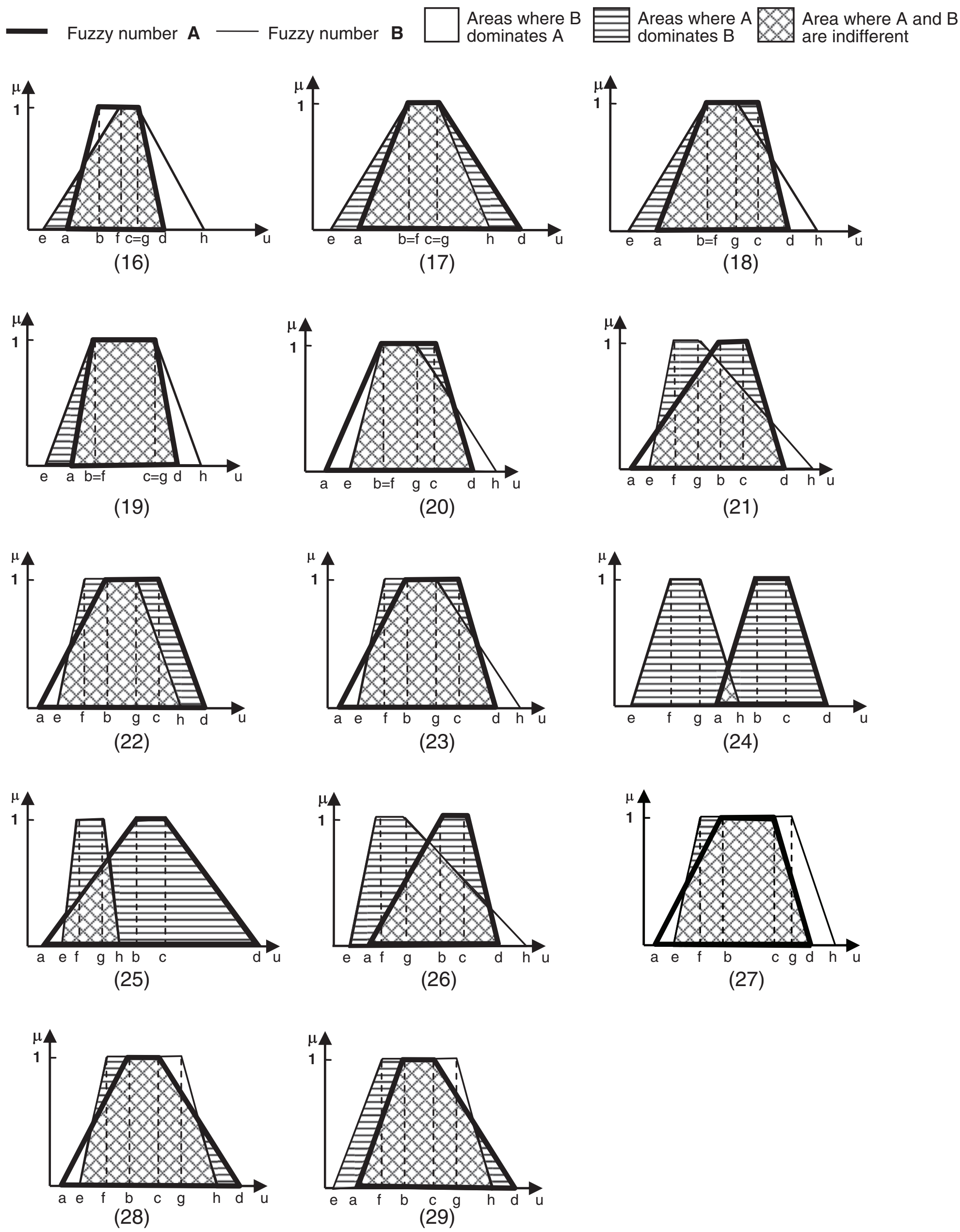
## Appendix 1. Twenty-nine cases of the general trapezoidal pairwise of fuzzy numbers




Fuzzy number A
Fuzzy number B
Areas where B dominates A
Areas where A dominates B
Area where A and B are indifferent
(16)
(17)
(18)
(19)
(20)
(21)
(22)
(23)
(24)
(25)
(26)
(27)
(28)
(29)