

International Conference
on Industrial Engineering and Systems Management

IESM' 2009

May 13 - 15, 2009

MONTREAL - CANADA

Iterative product configuration with fuzzy logic*

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Abstract

Product configuration provides an important opportunity for taking advantage of a number of the benefits of mass customization. Mass customization is aimed at developing a wide external variety of products to satisfy individual customers, with managed internal diversity to prevent cost proliferation. In this context, we propose an iterative product configuration method applying fuzzy logic which is designed to improve product configuration by replacing features which are of less interest to the customer with features the customer prefers. Fuzzy preference relations are used to evaluate the various configurations through the iterative product configuration process. To measure the level of customer satisfaction for each configuration, a satisfaction rate is also proposed. The integration of fuzzy preference relations and an adapted pseudo-order preference model constitute the basis for the proposed configuration method. An illustrative example is provided to show the applicability and practicality of the method.

Key words: product configuration, fuzzy preference, fuzzy ranking, fuzzy sets, mass customization.

1 Introduction

Manufacturers today are faced with fierce competition in the global marketplace, and, as a result, try to develop better and more accessible products for their customers. Their objective is to satisfy customer wants and needs without sacrificing efficiency, effectiveness, and profit [1]. Mass customization provides a way to achieve this, and product configuration is an important key to taking advantage of mass customization in the product development process. Many strategies have been proposed to make mass customization a reality, including modular design, delayed differentiation, platforms, modularity, and commonality, among others. Some of these strategies include fuzzy logic, in order to access more accurate information for their processes.

Fuzzy logic has the capacity to consider vague information related to human decisions. Recently, it has been applied in fields like decision-making and product development. In this vein, we propose an iterative method here for product configuration in which the fuzzy preference relation is applied to evaluating the relationship between different variables. A satisfaction rate is calculated to measure the change in the level of customer satisfaction as a result of the upgrade in product configuration.

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This paper is organized as follows: section 2 presents a literature review focusing on product configuration and fuzzy product configuration; section 3 describes the proposed method for product configuration and a detailed illustrative application; and section 4 concludes the paper and suggests some future research directions.

2 Literature review

This section focuses on the analysis of recent work on product configuration. The first part considers literature related to product configuration in a general way, and the second considers the application of fuzzy logic to product configuration.

2.1 Product configuration

The configuration a product is the representation of the logical and spatial arrangement of the various parts/subassemblies of that product with respect to one another considering the various kinds of constraints (e.g. technical, commercial) imposed on it [2]. Product configuration is an important area of opportunity for developing competitive products and is strongly correlated to mass customization because of the scope it provides for developing a large variety of products within the constraints and limitations of the manufacturer. Various approaches, models, and methods have been developed to achieve this. One of these is an approach designed to find configurations that match industry requirements, and consists of three steps: product configuration, bill of materials configuration, and routing configuration [3]. Another, which applies a design structure matrix to show the interaction flow between configuration elements, has been proposed to analyze the product configuration [4], and was designed to evaluate product configuration from the point of view of sales. Other approaches attempt to optimize the product configuration process, such as one based on a multi-objective genetic algorithm, which optimizes the design of the product configuration and focuses on the problem of combination explosion [5]. The models that have been proposed include a decision model to select concepts in product configuration by considering the interactions of the concepts caused by their constraints and functional couplings [6]. Also, an interesting application of the case-based reasoning algorithm has been presented to reduce the time and cost of the design process by generating the right bill of materials from the beginning of the product design process [7]. Similarly, a methodology and architecture designed to incorporate the requirement configuration and the engineering configuration into the configuration design process has been proposed [8]. This work integrates data mining approaches, such as fuzzy clustering and association rule mining to link customer groups with clusters of product specifications. Another product configuration method based on the multi-layer evolution model has been proposed as well [9], which considers the features of the customer requirements and the product configuration design analysis as performed in three layers: function, qualification, and structure, and also considers fuzzy and incomplete customer requirements. Even though fuzzy logic has been applied in some of the above work, these applications remain partial. In the following section, we analyze some work in which fuzzy logic is applied to product configuration in a more significant way.

2.2 Fuzzy product configuration

The application of fuzzy logic has been increasing in recent decades, and it has been used in interesting ways in issues related to product configuration, such as concept evaluation, design requirements, company capabilities, and customer requirements. Some of these uses are explained below.

A fuzzy ranking methodology has been developed to evaluate a conceptual design in the context of mass customization [10], in which a set of alternatives is evaluated and one is selected that can satisfy customer needs, considering the design requirements and technical capabilities of the company. Tsai and Hsiao [11] developed a method to translate customer needs into applicable alternatives to satisfy customer desires, applying fuzzy inference to establish the relationship between customer needs and product alternatives. Also, an integrated approach to designing configurable products based on multiple fuzzy models has been proposed [12], fuzzy methods such as fuzzy product specification, fuzzy functional network, fuzzy physical solution, and the fuzzy constraint model to translate the customer specifications into physical solutions dealing with various forms of uncertainty, such as imprecision, randomness, fuzziness, ambiguity, and incompleteness. Another approach concerns product-level configuration [13], which considers uncertain and fuzzy requirements provided by customers by applying fuzzy multi-attribute decision-making. More recently, this approach has been presented as

a method that can be used in a product data management system and on e-commerce websites, making it possible to obtain the customer's preferred product according to the utility value with respect to the whole set of product attributes [14]. In the same context, an iterative method applying fuzzy logic is proposed in the following section for product configuration with the objective of contributing to an increase in customer satisfaction by offering products that more closely match customer desires.

3 Iterative product configuration with fuzzy logic

3.1 General configuration process

This paper proposes a method for fuzzy product configuration, where the fundamental issue is the analysis of the fuzzy preference relation between some selected product features and customer preferences. To calculate the preference relation, we apply a method proposed by Tseng and Klein [15] and adapted by Barajas and Agard [16] in this work. Figure 1 depicts the proposed product configuration method.

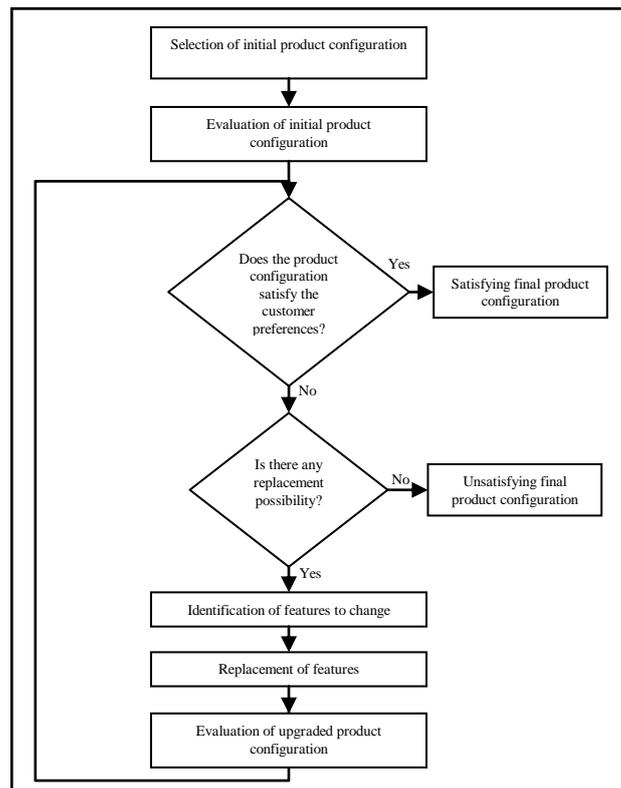


Fig. 1. Flow of the product configuration method

The configuration method depicted in Figure 1 starts with the selection of an initial product configuration that conforms to the cheapest alternative for each feature. This configuration should be evaluated, and this is achieved by assessing customer satisfaction. If the initial configuration does not satisfy customer requirements, potential improvements must be identified through an analysis of the replacement options. Then, if possible, all the identified features are replaced. The new configuration is evaluated and compared with the customer's preferences to confirm whether or not it satisfies those preferences.

3.2 Detailed configuration and application

Let us suppose that a laptop manufacturer aims to customize its production according to customer preferences by selecting from a list of configurable key features in an attempt to increase the compatibility between their products and those preferences, considering various criteria such as manufacturability, modularity, commonality, compatibility, functionality, and so on.

Suppose that the following five selected features are the most relevant for the laptop configuration: processor, operating system, display, memory, and hard drive. All these features and their alternatives are illustrated in Figure 2. There are three alternatives for the processor (F_{11}, F_{12}, F_{13}), two for the operating system (F_{21}, F_{22}), six for the display (F_{31}, \dots, F_{36}), four for memory (F_{41}, \dots, F_{44}), and six for the hard drive (F_{51}, \dots, F_{56}). Suppose that a cost-benefit analysis has been performed to prioritize the various alternatives for each feature based on value, and the versions are such that F_{ij+1} outperforms F_{ij} .

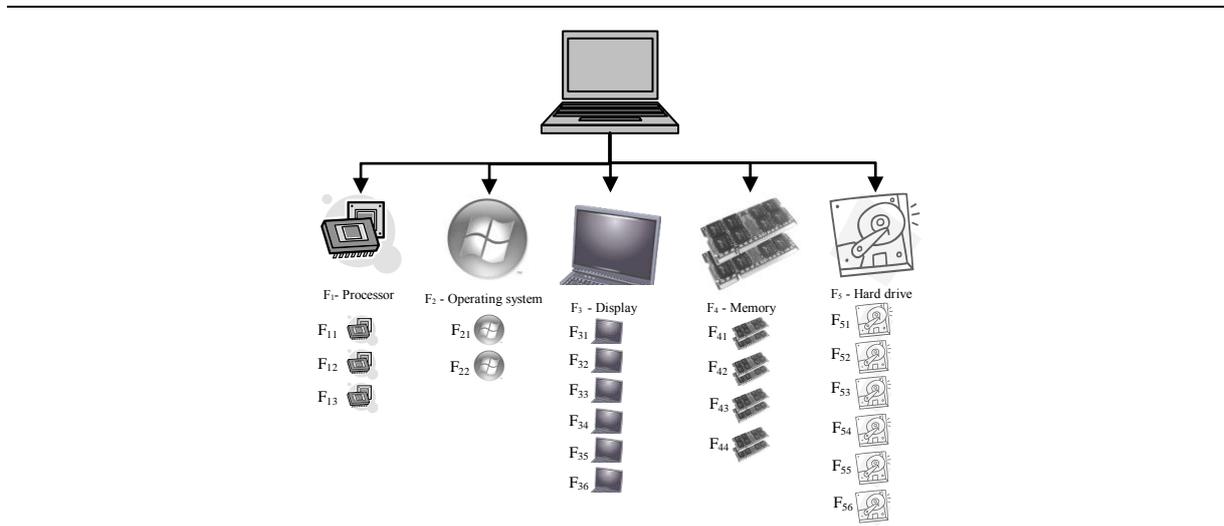


Fig. 2. Selected configurable features

3.2.1 Selection of initial product configuration

In our work here, the initial product configuration consists of the lowest-ranking option for each feature; that is, the lowest-ranking option in the hierarchical list of alternatives per feature ($F_{11}, F_{21}, F_{31}, F_{41},$ and F_{51}). This configuration constitutes the base on which to start the iterative process of feature substitution to reach the level of satisfaction demanded by the customer.

3.2.2 Evaluation of the initial product configuration

Selection of the best product for a customer based on a set of preferences is made possible by applying the following method [17] to determine the best product configuration. It consists of five steps:

- (1) *Market and technical evaluation of product features.* This evaluation can generally be performed by the industry concerned from specialized sources. If these are not available, a survey administered by experts can be used instead. This information must then be represented in fuzzy numbers. This fuzzification process should be performed by those with sufficient knowledge of the industry in question.
- (2) *General prioritization of features.* A customer survey can be used to obtain a general feature prioritization for the type of product in question.
- (3) *Customer preference consideration per each feature.* By posing a few questions phrased in colloquial or linguistic terms, it is possible to arrive at the customer preference for each feature. All these preferences should be represented by fuzzy numbers based on the general prioritization scale.
- (4) *Evaluation of product configuration.* Let $R(A,B)$ be the fuzzy preference relation and $\mu R(A, B)$ the membership function representation of $R(A,B)$. According to [15], if the membership degree $\mu R(A,B)$ is equal to 0.5, then A and B are indifferent.

To calculate the fuzzy preference relation $R(A,B)$, let A and B be two fuzzy numbers which are convex and normal. If there exists an area of overlap between fuzzy numbers A and B (intersection between A and B), then the overlap area is defined as the indifference area (see Figure 3). If there exist one or more non-overlap areas between fuzzy numbers A and B, then, for each non-overlap area, either A dominates B or B dominates A (see Figure 3).

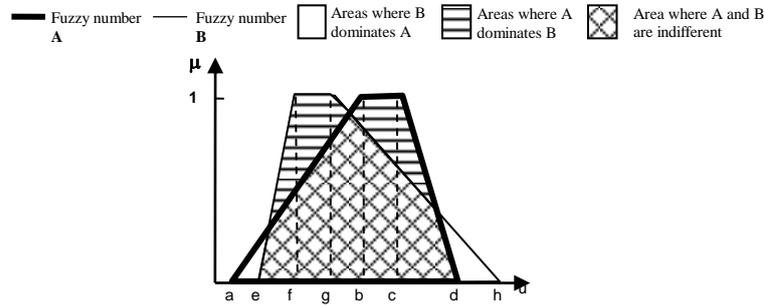


Fig. 3. Dominance and indifference between A and B

If A and B are two normal fuzzy numbers, then the fuzzy preference relations $R(A,B)$ or $R(B,A)$ could be obtained using the following equations:

$$R(A, B) = [D(A, B) + I(A, B)] / [A(A) + A(B)] \quad (1)$$

where $D(A,B)$ is the area in which A dominates B, $D(B,A)$ is the area in which B dominates A, $I(A,B)$ is the area in which A and B are indifferent, and $A(A)$ and $A(B)$ are the areas of A and B respectively.

Since $R(A,B)$ and $R(B,A)$ are reciprocal, that is, $R(A, B) + R(B, A) = 1$, then

$$R(B, A) = 1 - R(A, B) \quad (2)$$

$R(A,B)$ and $R(B,A)$ are interpreted as the degree to which A is preferred to B and B is preferred to A respectively.

To evaluate whether or not the initial product configuration satisfies the customer preference, the pseudo-order preference model can be applied, which has already been used in the literature several times [18], [19], [20], [21]. Let the fuzzy preference relation between two ideas A and B for criterion i be obtained by the pairwise comparison of $g_i(A)$ and $g_i(B)$ represented by fuzzy numbers. Three types of preference relation are defined in terms of the fuzzy preference relations between these two alternatives $\forall a, b \in A$ and $i \in C$, as follows:

$$AP_iB \Leftrightarrow P(g_i(A), g_i(B)) - P(g_i(B), g_i(A)) > p_i,$$

$$AQ_iB \Leftrightarrow P(g_i(A), g_i(B)) - P(g_i(B), g_i(A)) \leq p_i,$$

$$AI_iB \Leftrightarrow |P(g_i(A), g_i(B)) - P(g_i(B), g_i(A))| \leq q_i,$$

where P_i and Q_i depict a strict and a weak preference respectively, and I_i depicts an indifference relation. The preference threshold p_i and the indifference threshold q_i (defined by common sense [18]) are used to discriminate between the indifference, strict preference, and weak preference of two alternatives for criterion i. The three possible types of preference should be read as follows:

- AP_iB , where there is a strict preference between ideas A and B (idea A is strictly preferred to idea B for criterion i)
- AQ_iB , where there is a weak preference between ideas A and B (idea A is weakly preferred to idea B for criterion i)
- AI_iB , where there is no difference between ideas A and B (idea A is not different from idea B for criterion i).

If A represents the product feature and B represents the customer preference, the above types of preference can be applied as follows: A/I_iB represents the case where the product feature satisfies the customer preferences fairly. This situation represents the principal target in this work. Cases AP_iB and AQ_iB represent the situations where the product feature exceeds customer preferences, and cases BP_iA and BQ_iA where the product features fall short of the customer preferences, and these situations do not correspond to the target. If the fuzzy preference relation for all the pairwise combinations of product features and customer preferences correspond to the case $a/I_i b$, this means that all of them can be part of a possible product configuration if that case satisfies a fixed percentage of customer satisfaction.

Let us apply all these steps to evaluate the initial product configuration:

- (1) *Market and technical evaluation of product features.* Let us suppose that a group of experts in the industry in question evaluated each feature. The values for each are represented by fuzzy numbers as follows:

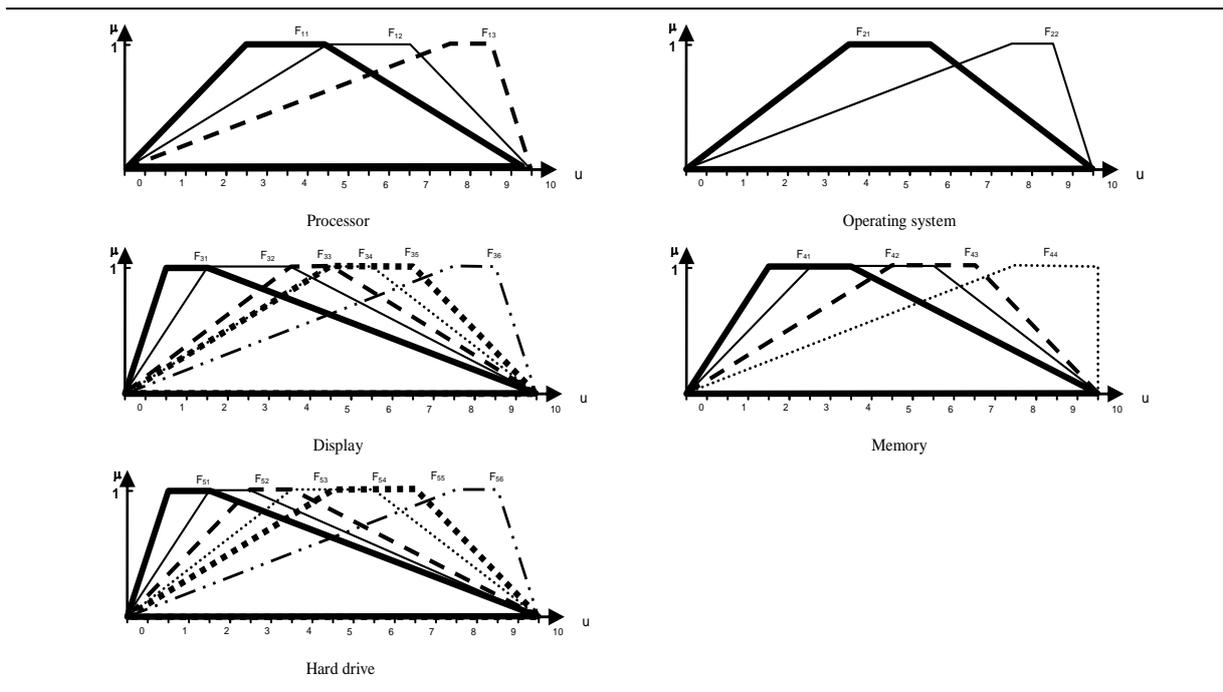


Fig. 4. Customer satisfaction based on a feature's cost-benefit relation

Figure 4 depicts the fuzzy representation of the cost-benefit relation evaluation for each of the selected product's features.

- (2) *General prioritization of features.* In the same way, a general feature prioritization has been created by using a survey to define customer preferences relating to the product in question. These preferences are expressed in colloquial terms, such as *not important*, *less important*, *moderately important*, *important*, and *highly important*, as is depicted in Figure 5.

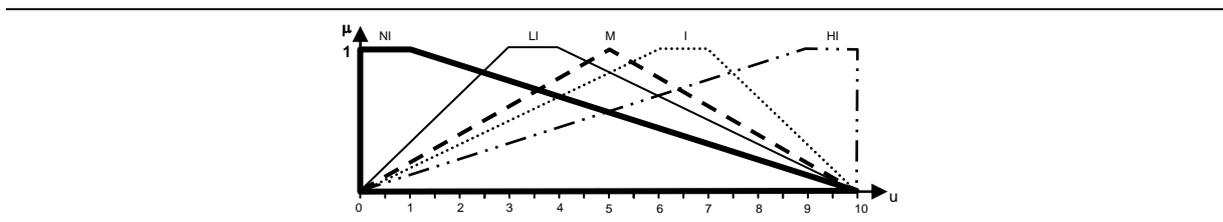


Fig. 5. A feature's general prioritization

The respective fuzzy number for each level of the general prioritization scale shown in Figure 5 is listed in Table 1.

Table 1 Feature’s prioritization representation

Level of prioritization	Fuzzy number representation
HI- ‘Highly Important’	[1 9 10 10]
I- ‘Important’	[1 6 7 9]
M- ‘Moderately important’	[1 5 5 9]
LI- ‘Less Important’	[1 3 4 9]
NI- ‘Not Important’	[0 0 1 9]

(3) *Customer preference consideration for each feature.* The customer preferences for each feature are listed in Table 2, and these preferences are expressed in linguistic or colloquial terms.

Table 2 Feature preferences per customer

Product features	Customer preference per feature
F ₁ , Processor	HI
F ₂ , Operating system	HI
F ₃ , Display	HI
F ₄ , Memory	HI
F ₅ , Hard drive	HI

(4) *Evaluation of product configuration.* The issue of the fuzzy preference relation is fundamental to the evaluation process, and equation 1 can be applied to calculate such a relation. Let us consider R(A, B) as the fuzzy preference relation between product features (A) and customer preferences (B). If the membership function $\mu_R(A, B)$ is equal to 0.5, then there is no significant difference between them. This situation is displayed in Table 3, where F_{ij} represents the set of features (i) for each configuration (j), and C_{ki} represents the set of customer preferences (k) for each feature (i). If the fuzzy preference relation for all the pairwise combinations between F_{ij} and C_{ki} are equal to 0.5, then a possible satisfactory configuration can be obtained if this configuration satisfies the minimum level of customer satisfaction.

Table 3 Indifference fuzzy preference

F _{ij} \C _{ki}	C _{ki+1} - F _{ij+1} I _i C _{ki+1}	C _{ki+2} - F _{ij+2} I _i C _{ki+2}	C _{ki+3} - F _{ij+3} I _i C _{ki+3}	C _{ki+4} - F _{ij+4} I _i C _{ki+4}
F _{ij}	0.5 - F _{ij} I _i C _{ki}			
F _{ij+1}		0.5 - F _{ij+1} I _i C _{ki+1}		
F _{ij+2}			0.5 - F _{ij+2} I _i C _{ki+2}	
F _{ij+3}				0.5 - F _{ij+3} I _i C _{ki+3}
F _{ij+4}				

3.2.3 Evaluation of customer satisfaction

Customer satisfaction (CS) is evaluated using equation 4:

$$CS_j = \left[\frac{\sum_{j=1}^m R(A_{ij}, B_{ki}) / m}{0.5} \right] \times 100 \tag{4}$$

where:

R(A_{ij}, B_{ki}) is the fuzzy preference relation between A_{ij} and B_{ki}.

A_{ij} = {A₁₁, A₂₁, ..., A_{mn}} is the set of features (i) for each configuration (j) $\forall i \in [1, n]$, and $\forall j \in [1, m]$.

B_{ki} = {B₁₁, B₁₂, ..., B_{pn}} is the set of customer preferences (k) for each feature (i) $\forall k \in [1, p]$, and $\forall i \in [1, n]$.

Once a possible product configuration has been found, it is necessary to evaluate the level of customer satisfaction for such a configuration. This evaluation can be obtained by applying Equation 4. If the percentage of customer satisfaction is less than the level fixed for acceptance, then replacement features should be considered, if they are available. For this application, six different evaluations have been performed (see Table 4 and Figure 6).

3.2.4 Replacement possibilities analysis

If the percentage of customer satisfaction does not match customer expectations, then it is necessary to check whether or not other features are available for replacement. To perform this evaluation, all product features must be listed hierarchically, such that the first option belongs to the lowest ranking option for each feature. For example, if there exist five options for feature 1 (A_1), a hierarchical code can be expressed as (A_{ij}) , where (i) identifies the feature and (j) identifies the hierarchical precedence as $A_{11}, A_{12}, A_{13}, A_{14}, A_{15}$. For this application, there exist five different options for feature 1 (F_1), and their hierarchical codes are expressed as (F_{ij}) , where (i) and (j) identify the feature and the hierarchical precedence as $F_{11}, F_{12}, F_{13}, F_{14}, F_{15}$ respectively.

3.2.5 Identification of features to change

If the hierarchical precedence of feature (A_{ij}) in the current product configuration is less than the maximum A_{ij} ($j < j_{max}$), then there exists a replacement opportunity for this feature. An evaluation should be performed for each feature.

3.2.6 Replacement of features

Once all the replacement opportunities for each feature have been identified, they must all (A_{ij}) be replaced by the next feature (A_{ij+1}) .

3.2.7 Evaluation of upgraded product configuration

For each replacement iteration, the upgraded configuration must be evaluated by applying the procedure explained in step 3.2.2.

3.2.8 Verify whether or not the final product configuration satisfies the customer preferences

If the percentage of customer satisfaction is greater than or equal to the acceptance percentage fixed by the customer, then the new product configuration satisfies its preferences. If not, an unsatisfactory product configuration is obtained. For this application, let us consider a minimum level of customer satisfaction of 90%.

Table 4 Product satisfaction rate per product configuration

Number of iterations	Configuration improvement by configuration per customer
1	58.788
2	68.452
3	75.96
4	82.856
5	84.476
6	91.852

Table 4 displays the changes in the customer satisfaction percentage for all six possible iterations to obtain a new product configuration. Figure 6 shows the behavior of this iterative process graphically. Appendix 1 depicts the fuzzy preference relations for the first three iterations used to obtain the customer satisfaction percentages.

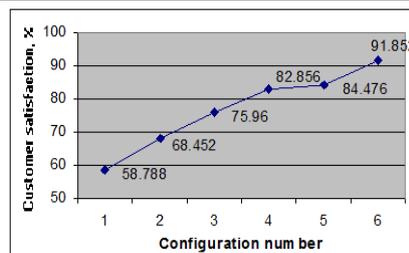


Fig. 6. Configuration improvement per customer

Table 4 and Figure 6 show that the best product configuration for the customer is made up of the features (F_{13} - F_{22} - F_{36} - F_{44} - F_{56}), according to the nomenclature used in Figure 2.

4 Conclusions

Product configuration is a key issue in the development of better products aimed at increasing the level of customer satisfaction. In our work here, fuzzy logic has been applied to enrich this issue. We are proposing a method to configure a suitable product for a specific customer. Fuzzy preference relation analysis has been used for the evaluation of the product configurations. Also, a metric to measure the customer satisfaction for each configuration is proposed. Fuzzy preference relation and an adapted pseudo-order preference model have been applied as principal tools into the proposed method. The application presented in section 3 reveals the practical applicability of fuzzy logic in the various areas, like the configuration of modular and scalable products. The output of the proposed method is a personalized product considering the preferences of a specific customer. This method contributes at allowing forming product closer to the customer preferences increasing the customer satisfaction. Some future research directions could include the integration of fuzzy logic in a general methodology to design families of products.

5 Acknowledgments

This research was supported by funding from the Natural Sciences and Engineering Research Council of Canada (NSERC) and by the Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT).

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Appendix 1: Fuzzy preference relation per feature

F _{ij} C _{ki} – type of relationship	C ₁₁	C ₁₂	C ₁₃	C ₁₄	C ₁₅
	[0 9 10 10]	[0 9 10 10]	[0 9 10 10]	[0 9 10 10]	[0 9 10 10]
F ₁₁ [0 3 5 10]	0.3106 – C ₁₁ Q ₁ F ₁₁				
F ₂₁ [0 4 6 10]		0.3344 – C ₁₂ Q ₁ F ₂₁			
F ₃₁ [0 1 2 10]			0.2674 – C ₁₃ Q ₁ F ₃₁		
F ₄₁ [0 2 4 10]				0.2899 – C ₁₄ Q ₁ F ₄₁	
F ₅₁ [0 1 2 10]					0.2674 – C ₁₅ Q ₁ F ₅₁
F ₁₂ [0 5 7 10]	0.3623 – C ₁₁ Q ₂ F ₁₂				
F ₂₂ [0 8 9 10]		0.4545 – F ₂₂ I ₂ C ₁₂			
F ₃₂ [0 2 4 10]			0.2899 – C ₁₃ Q ₂ F ₃₂		
F ₄₂ [0 3 6 10]				0.3205 – C ₁₄ Q ₂ F ₄₂	
F ₅₂ [0 2 3 10]					0.2841 – C ₁₅ Q ₂ F ₅₂
F ₁₃ [0 8 9 10]	0.4545 – F ₁₃ I ₃ C ₁₁				
-----		0.4545 – F ₂₂ I ₃ C ₁₂			
F ₃₃ [0 4 5 10]			0.3247 – C ₁₃ Q ₃ F ₃₃		
F ₄₃ [0 5 7 10]				0.3623 – C ₁₄ Q ₃ F ₄₃	
F ₅₃ [0 3 4 10]					0.303 – C ₁₅ Q ₃ F ₅₃