

# Product and supply chain design using a two-phases optimisation approach

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## Abstract

When designing a new product family, designers and manufacturers have to define simultaneously the product structure and its supply chain. This leads to a complex optimization problem to solve in order to satisfy diversified customers' requirements with various options and variants. The paper focuses on modeling and solving this design problem. It consists in selecting a set of modules that will be manufactured in distant facilities and shipped in a nearby location plant for a final assembly operation under time limits. The objective is to determine the optimal set of modules able to define the bill of material of each finished product and to minimize assembly and production costs. We propose in this paper a two phase approach and an integrated one to solve this problem. We provide experiments on small instances in order to better understand the links between the product design and the supply chain design.

**Keywords:** product family design, supply chain design, optimization.

## 1 Introduction

Nowadays, the growing demand for customized products involves an increasing number of product variants and options. It follows a complex product diversity to manage. This variety must be controlled in term of product, process and supply chain costs, as well as customer lead-time. Consequently, when designing a new product family, a consistent approach is necessary to quickly

define a set of variants and the relevant supply chain, in order to guarantee the customers' satisfaction and to minimize the total investment and operating cost of the global supply chain [?].

In order to give an efficient answer to this problem without an expansive product proliferation, companies may focus on "mass customization" [17]. Mass customization deals with large products portfolio, flexible manufacturing systems and extended supply chain.

A product family is composed by similar products that differ by some characteristics such as options. For example, the basic car model may contains few options in order to minimize the sale price. Then, some options can be added to this basic model like air-conditioning, automatic gear box or diesel engines and so on.

There are two extreme production strategies that a company can use. The first one consists to make to stock the different products. This leads to select a minimum set of standardized products [3], that could include supplementary options to meet diversified customer requirements. However, storage costs may be too high because of the large product portfolio. The second strategy consists to produce only when an order is received. In this case, the lead time may be higher leading to the non satisfaction of the customer. An intermediate strategy consists to manufacture pre-assembly components, called modules, for stock and to assemble them when an order is planed. The advantage of such strategy is to reduce the lead time and to avoid great storage costs.

In this paper, we explore this production policy where modules are manufactured in distant location facilities for cost minimization. Those modules are shipped and assembled in a nearby location facility in order to have a short lead-time for the customers. We present a comparison between two modeling strategies: (1) a two phases approach in which costs are optimized separately between the nearby and the distant facilities and (2) an integrated approach which takes into account simultaneously the process and the supply chain costs. Electric beam is an example of this category of product family. They are largely used in the car industry [11]. In section 2 we give an overview on the state of art of the different studies related to the problem at hand. We give a more detailed description of the problem in section 3. An Integer Linear Program model is given in section 4 and some computational experiments are given and analyzed in section 5. Finally concluding remarks and perspectives are proposed in section 6.

## 2 State of the art

Literature proposes various approaches for the design of product families. Some product design methodologies focus on the product architecture [4],[9], this is advantageously supported with modular design [8], component / product / process standardization [10], [15],[18]. Different methodologies concentrate on process standardization [15], process resequencing [13] or generic assembly routing [6],[7]. From another point of view, authors concentrate on supply

chain design methodologies, by integrating customers and suppliers in uncertain environment [5], centered on stocks management [14] and for distant facility location [16].

In all these works, the design of product, process and supply chain are integrated two by two. However, Lee [12] shows that the product design choices (modular design, standardization or delayed differentiation) have a strong influence on the supply chain design. Hence it is important to determine simultaneously the product family design and its associated supply chain.

There is some recent works dealing with a global design modeling. Agard et al. [1] propose a genetic algorithm to minimize the mean of finished product assembly times for a given demand. Agard and Penz [2] propose a model for minimizing module production costs and a solving approach based simulated annealing. Lamothe et al. [11] use a generic bill of material representation in order to identify simultaneously the best bill of material for each product and the optimal structure of the associated supply chain.

### 3 Problem Presentation

Consider the following industrial context (Figure 1). The producer receives customers' orders for finished products containing options and variants. Each individual product is then manufactured from a set of modules that come from various suppliers.

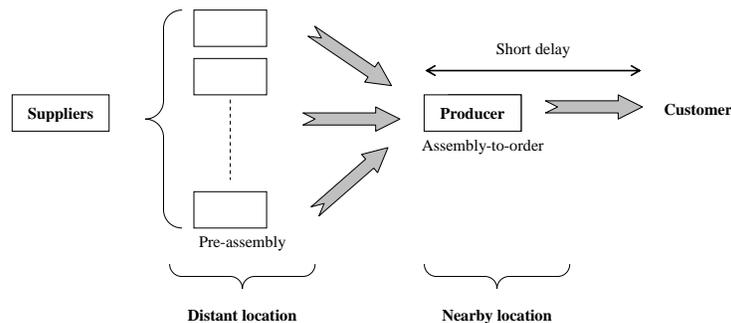


Figure 1: Structure of the supply chain

Consider now that the producer has only a short delay ( $T$ ) to respond to customer demands. This delay is less than the necessary time to assemble products from elementary components. In addition to this, the producer has to provide the product exactly like the customer demand (without extra options). This constraint comes from technical considerations or simply to avoid supplementary costs.

To satisfy customers, the producer brings pre-assembled components, called modules, from many suppliers which are located in distant facilities around the world. The suppliers' facilities are characterized by a very weak production costs. Then, the modules are assembled in the producer facility which we assume to be very close to the customers and thus characterized by its great reactivity and a reduced lead-time.

## 4 Model and solution approach

### 4.1 Notations

Specifying the problem assumptions: a product or a module is considered as the set of functions that it must fill, then:

- a function  $F_k$  is a requirement that must be ensured by the finished product.
- a module  $M_j$  is an assembly of functions that could be added with other modules to make a finished product.
- a finished product  $P_i$  is an assembly of modules that corresponds exactly to at least one customer demand.

Let introduce the following notations:

- $F = \{F_1, \dots, F_q\}$  : set of  $q$  functions that can appeared in both finished products and modules;
- $P = \{P_1, \dots, P_n\}$  : set of  $n$  possible finished products that may be demanded by at least one customer. We note  $D_i$  the estimated demand of the product  $P_i$  during the life cycle of the product family.
- $M = \{M_1, \dots, M_m\}$  : set of  $m$  possible modules.
- $S = \{S_1, \dots, S_s\}$  : set of  $s$  distant production facilities where a site  $S_l$  has a production capacity  $Cap_l$ .
- $CF_j$  : the management fixed cost of module  $M_j$  in the nearby facility.
- $CV_j$  : the assembly variable cost of module  $M_j$  in the nearby facility.
- $CF_{jl}$  : the management fixed cost of module  $M_j$  in facility  $S_l$ .
- $CV_{jl}$  : the production variable cost of module  $M_j$  in facility  $S_l$ .
- $w_j$  : the necessary time to assemble the module  $M_j$  in a finished product.
- $C_{jl}$  : the load caused by producing module  $M_j$  in facility  $S_l$ .

Under these assumptions, we can represent a product (or a module) by a binary vector of size  $q$ . Each element shows whether the corresponding function is required in the product (value = 1) or not (value = 0). The set  $M$  contains  $m$  modules. It may be a selection of modules defined by the engineering or all the possible modules issued from the whole combinatory.

The problem is now to determine the subset  $M' \in M$ , of minimum cost, such that all products in  $P$  can be built in a constrained time window  $T$  with elements from  $M'$ . Concerning the products, the goal is to determine which bill of material is the most suitable (Figure 2).

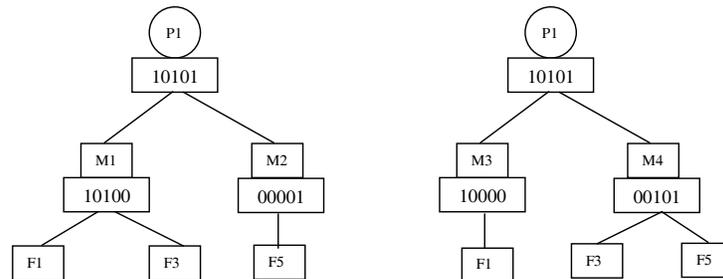


Figure 2: alternative bills of material

In order to take into account the process design, (1) the producer assembly line costs are considered - (2) finished product assembly time must be less than the available time to respect the time delivery for the customers. In order to take into account the supply chain design, distant location production costs are considered (transportation costs are implicitly considered in the production costs because we treat a problem with only one nearby location facility).

We have done a model of the problem using an Integer Linear Program formulation. Our objective consists in minimizing all the costs linked to the activity of the producer and suppliers. These costs are : fixed costs due to modules management in the nearby location facility, modules assembly costs in the nearby location facility, fixed costs due to modules management in the distant location facilities and modules production costs in the distant location facilities.

In the following, we distinguish two strategies for solving the problem:

- A two-phases approach,
- An integrated approach.

The two following sections present these strategies in more details.

## 4.2 A two-phases approach

The basic idea in the first approach is to optimize costs separately between the nearby location facility and the distant location facilities.

The first phase consists in determining modules which optimize assembly costs in nearby location facility such that all finished products can be build in the constrained time window  $T$ :

$$Z1 = \min \sum_{j=1}^m CF_j Y_j + \sum_{j=1}^m CV_j \left( \sum_{i=1}^n D_i X_{ij} \right)$$

s.t.

$$AX_i = P_i \quad \forall i \in \{1, \dots, n\} \quad (1)$$

$$\sum_{j=1}^m w_j X_{ij} \leq T \quad \forall i \in \{1, \dots, n\} \quad (2)$$

$$X_{ij} \leq Y_j \quad \forall i \in \{1, \dots, n\} \quad \forall j \in \{1, \dots, m\} \quad (3)$$

$$Y_j, X_{ij} \in \{0, 1\} \quad \forall i \in \{1, \dots, n\} \quad \forall j \in \{1, \dots, m\} \quad (4)$$

Where  $X_{ij} = 1$  if module  $M_j$  is used in the bill of material of product  $P_i$ , 0 otherwise.  $Y_j = 1$  if module  $M_j$  is selected (belongs to  $M'$ ), 0 otherwise.  $A$  is the binary matrix whose the column  $j$  is the vector  $M_j$ .  $X_j$  is the column vector composed by the variables  $X_{ij}$ .

The objective function minimizes the costs occuring in the nearby location facility, where  $(\sum_{i=1}^n D_i X_{ij})$  is the total demand of module  $M_j$ .

Constraint (1) shows that a finished product  $P_i$  must be assembled exactly like the customer demand. Constraint (2) indicates that products must be assembled within the time window  $T$  in order to respect the delivery time. Constraint (3) traduces the relation between  $X_{ij}$  and  $Y_j$  variables. If a module is used in the bill of material of some products then it belongs to  $M'$ .

The problem described here contains the set partitioning problem. We then conclude that it is NP-hard in the stong sense.

The second phase deals with the assignment of modules issued from the first phase on the distant location facilities under capacity constraints:

$$Z2 = \min \sum_{l=1}^s \sum_{j|Y_j=1} CF_{jl} Y_{jl} + \sum_{l=1}^s \sum_{j|Y_j=1} CV_{jl} \left( \sum_{i=1}^n D_i X_{ij} \right) Z_{jl}$$

s.t.

$$\sum_{l=1}^s Z_{jl} = 1 \quad \forall j|Y_j = 1 \quad (5)$$

$$\sum_{j|Y_j=1} C_{jl} \left( \sum_{i=1}^n D_i X_{ij} \right) Z_{jl} \leq Cap_l \quad \forall l \in \{1, \dots, s\} \quad (6)$$

$$Z_{jl} \leq Y_{jl} \quad \forall j|Y_j = 1 \quad \forall l \in \{1, \dots, s\} \quad (7)$$

$$Z_{jl} \geq 0 \quad \forall j|Y_j = 1 \quad \forall l \in \{1, \dots, s\} \quad (8)$$

$$Y_{jl} \in \{0, 1\} \quad \forall j|Y_j = 1 \quad \forall l \in \{1, \dots, s\} \quad (9)$$

Where  $Y_{jl} = 1$  if module  $M_j$  is produced in facility  $S_l$ , 0 otherwise.  $Z_{jl}$  is the percentage of demand of module  $M_j$  produced in facility  $S_l$ .

The objective function minimizes the costs occuring in all distant location facilities. Constraint (5) indicates that the production of a module  $M_j$  must satisfy

the requirements. Constraint (6) shows that production in facility  $S_l$  must not exceed its capacity. Constraint (7) expresses the relation between  $Z_{jl}$  and  $Y_{jl}$  variables. A module  $M_j$  can be produced in  $S_l$  only if  $M_j$  is assigned to  $S_l$  ( $Y_{jl} = 1$ ).

Many variants can affect constraint (7):

- a module  $M_j$  could be produced in many distant location facilities, this is the case of the model described above.
- the production of a module  $M_j$  is restricted in only one facility, in this case we have to add the following constraint:  $\sum_{l=1}^s Y_{jl} = 1 \forall j | Y_j = 1$
- every module must be produced in at least two facilities with a minimum percentage of  $\delta$  in each one. This is to anticipate production problems like delivery delay or worker strikes. Hence we have to add the following two constraints:  $Z_{jl} \geq \delta Y_{jl} \forall j | Y_j = 1 \forall l \in \{1, \dots, s\}$  and  $\sum_{l=1}^s Y_{jl} \geq 2 \forall j | Y_j = 1$

### 4.3 An Integrated approach

The second strategy consists in optimizing all costs at the same time, where the objective function is the sum of the two phase objective functions (second phase function will be extended on all modules). Constraints are those of the two phases. The idea of such approach is to make a global decision view when designing both the process and the supply chain. However, the number of variables is much higher with this approach compared to the previous one. Then it can be used for small instances but it is not suitable for large ones.

## 5 Computational experiments

### 5.1 Data sets and experimental conditions

Our objective was to analyze the optimal solution behavior for several cost configurations and for different time windows  $T$ . For this aim, we have done the tests on small instances on which the module set, the finished product set, the distant facility set, the demand  $D_i$ , the assembly operating times  $w_j$ , the distant facility capacities, the distant facility costs and the nearby facility variable costs are fixed while the nearby facility fixed costs was increased progressively. In order to simplify the problem data, we introduced some rules on the different costs occurring in the first optimization phase:

- $CF_j = \alpha(f(q_j) + \delta_1)$ ,
- $CV_j = \beta(f(q_j) + \delta_2)$ .

Where  $q_j$  is the number of existing functions in module  $M_j$ .  $\alpha$  and  $\beta$  are coefficients such that  $\alpha \geq \beta$  and  $\delta_1, \delta_2$  are jamming factors.  $f$  is a non decreasing

function.

We have tested three scenarios for the cost function  $f$ :

- The square root function  $f(q_j) = \sqrt{q_j}$
- The identity function  $f(q_j) = q_j$
- The square function  $f(q_j) = (q_j)^2$

The problem data was fixed as follow:  $q = 8$ ,  $n = 30$ , where each product has at least 3 functions and at most 6,  $m = 255$  (all possible combinatory of modules) and  $s = 2$ . The distant location facility costs and loads are randomly generated.  $\beta = 10$ ,  $\delta_1 = 10\%$ ,  $\delta_2 = 8\%$ , and  $\alpha$  is given by the expression  $50\eta$  where  $\eta \in \{1, 20, 40, 60, 80\}$ .  $T$  varied from 2 to 6 (the maximum number of functions in a finished product).

The tests have been done in C++ with Ilog Cplex9.0 library. They have been solved in a DELL station / 2.8 GHZ/ 1Go RAM.

## 5.2 Result analyses

Figure 3, 4 and 5 show the numerical results for different tests: the total fixed costs in the nearby location facility ( $TotCF$ ), (the total modules assembly costs in the nearby facility ( $TotCV$ ), the total costs in the nearby location facility (objective function  $Z1$ ), the total costs in the distant location facilities (objective function  $Z2$ ), the producer and supplier total costs ( $Z1 + Z2$ ), the number of modules in  $M'$  for the two-phase approach (Size1), the computational time of resolution of the two-phases model approach, the objective optimal value of the integrate model approach, the number of modules in  $M'$  for the integrated approach (Size2), the computational time for the integrated model, and the gain percentage between the two-phase approach and the integrated one.

Concerning the square root function, for  $\eta = 1$ , the fixed costs are very neglectable compared to the variable costs, and since that there is no great gap between variable costs of small modules (those who have a small number of functions) and big ones, then optimal solution consists of a unitary bill of materials (only one component in the bill of materials of a product which is itself). Otherwise we will have modules with great requirements which leads to a greater objective function.

As soon as the fixed costs increase, we attend a balance between fixed and variable costs, which leads to a solution with less modules. The optimization of fixed costs favors the selection of small modules in order to minimize the number of modules, while the optimization of variable costs favors the selection of big modules in order to limit the demand quantity. We note also that increasing  $T$  allows to minimize the number of modules (for a balanced costs configurations) by favoring small modules selection that will be used in many bills of materials.

For the identity function, we remark that optimal solution has got always the same value of the total variable costs for any fixed cost configuration. Indeed, as the assembly variable cost of each module is equal to the number of its

Data		f = Square root function										gain perc.
η	T	Two-phases Approach					integrate approach					
		Tot CF	Tot CV	Z1	Z2	Objective function	Size1	Comp. Time (sec.)	Objective function	Size2	Comp. Time (sec.)	
1	2 to 6	3,4	52,8	56,2	10,5	66,7	30	219	66,7	30	0,3	0,00%
20	2	34,2	66,3	100,5	13,9	114,4	19	430	114,1	19	514	0,22%
	3 to 6	26,9	71,7	98,6	15,5	114,1	16	549	113,1	17	426,8	0,94%
40	2	59,6	72,7	132,3	14,4	146,7	19	2319	146,3	19	2416	0,31%
	3	41,1	79,7	120,8	16,7	137,6	14	459	136,5	14	422	0,79%
	4	33,0	86,0	118,9	21,8	140,7	12	76	135,1	13	128	4,01%
	5 to 6	30,0	88,3	118,2	20,9	139,2	11	73	135,1	13	96	2,94%
60	2	89,3	72,7	162,1	14,4	176,5	19	2597	176,5	19	4308	0,00%
	3	54,3	85,2	139,4	21,8	161,2	13	349	155,8	14	313	3,37%
	4	41,5	91,6	133,1	25,8	158,9	11	10	153,3	11	46	3,54%
	5 to 6	40,4	91,5	131,9	22,0	153,9	11	6	152,6	11	36	0,85%
80	2	119,1	72,7	191,9	14,5	206,4	19	2501	206,3	19	3229	0,05%
	3	72,3	85,2	157,5	21,8	179,3	13	122	174,3	14	110	2,80%
	4	53,9	92,9	146,8	22,2	168,9	11	5	168,4	11	14	0,27%
	5	47,7	96,5	144,1	25,6	169,7	10	3	166,1	11	10	2,13%
	6	41,4	101,9	143,3	25,1	168,4	9	3	166,1	11	9	1,36%

Figure 3: Results for  $f(q_j) = \sqrt{q_j}$

functions, then the assembly of each finished product will generate a variable cost equal to the number of the product functions multiplied by its demand. This is independently of the product bill of material, since the bill of material contains exactly the same number of the product functions. Hence, the optimization of such as function gets back to optimize only fixed costs and then to favors small number of modules.

Square root function favors always small number of modules for any balance report between fixed and variable costs, because of the extremely great gap between variable costs for small and big modules.

Computational time is very weak for the integrated model, this permits to conclude that the second phase seems to be not very difficult (may be because we tested an instance having only two distant location facilities). We remark also that there is not a clear improvement in the global objective function by optimizing integrated model. This is due to the heavy part of first phase costs in the global objective function. Other instances are tested now. The objective of these tests is to validate the results on more instances. We test also situations when costs of the two phases are equivalent or when costs of the second phase are higher than those of the first one. The initial findings of these tests reveal that the integrated approach is greatly more valuable when production costs are more preponderant than assembly costs, and specially when production variable costs are more important than production fixed costs. In this precise case, the gain percentage could reach up to 50%.

Data		f = Identity function										
η	T	Two-phases Approach						integrate approach			gain perc.	
		Tot CF	Tot CV	Z1	Z2	Objective function	Size1	Comp. Time (sec.)	Objective function	Size2		Comp. Time (sec.)
1	2	2,2	108,7	110,9	14,4	125,3	19	1068	121,8	24	5	2,80%
	3	1,2	108,7	109,8	21,5	131,3	14	90	121,1	20	4,5	7,77%
	4	0,8	108,7	109,5	28,4	137,8	11	3	121,1	20	4,5	12,15%
	5	0,6	108,7	109,2	30,6	139,8	9	0,2	121,1	20	4,1	13,39%
	6	0,4	108,7	109,1	31,9	141,0	8	0,2	121,1	20	3	14,13%
20	2	44,0	108,7	152,7	14,5	167,2	19	1750	167,1	19	825	0,06%
	3	23,1	108,7	131,8	21,5	153,3	14	90	147,8	14	35	3,61%
	4	15,4	108,7	124,1	28,2	152,3	11	3	144,4	12	8	5,20%
	5	11,0	108,7	119,7	30,8	150,5	9	0,2	142,2	10	4,2	5,51%
	6	8,8	108,7	117,5	31,9	149,4	8	0,2	142,2	10	2,7	4,82%
40	2	88,0	108,7	196,7	14,5	211,2	19	1638	211,1	19	739,9	0,05%
	3	46,2	108,7	154,9	26,0	180,9	14	137	170,9	14	41,4	5,56%
	4	30,8	108,7	139,5	23,9	163,4	11	3	160,2	11	3,8	1,98%
	5	22,0	108,7	130,7	30,8	161,5	9	0,3	155,4	10	2,7	3,77%
	6	17,6	108,7	126,3	31,9	158,2	8	0,2	155,4	10	2,8	1,77%
60	2	132,0	108,7	240,7	14,4	255,1	19	1855	255,1	19	2616	0,00%
	3	69,3	108,7	178,0	18,8	196,8	14	105	194,0	14	85	1,45%
	4	46,2	108,7	154,9	26,0	180,9	11	3	175,6	11	3,2	2,91%
	5	33,0	108,7	141,7	30,8	172,5	9	0,3	168,6	10	0,3	2,25%
	6	26,4	108,7	135,1	31,9	167,0	8	0,2	166,8	9	0,25	0,12%
80	2	176,0	108,7	284,7	14,5	299,2	19	1893	299,1	19	2460	0,03%
	3	92,4	108,7	201,1	20,2	221,3	14	164	217,1	14	79,8	1,89%
	4	61,6	108,7	170,3	26,0	196,3	11	3	191,0	11	3,6	2,68%
	5	44,0	108,7	152,7	30,8	183,5	9	0,3	181,8	10	1	0,92%
	6	35,2	108,7	143,9	31,9	175,8	8	0,2	175,8	8	0,3	0,00%

Figure 4: Results for  $f(q_j) = q_j$

## 6 conclusion

This paper was dedicated to a difficult industrial problem arising when companies try to offer a large variety of products to consumers. In this problem, a choice of components (modules) has to be efficient. These modules are produced for stock, and used in the last stage, in the assembly line. Several authors considered this problem, using different assumptions - a function can appear twice in a final product, a final product can be substituted by another one containing more functions - but few papers consider the problem in which each final product must correspond exactly to the demand.

We presented a new challenging model that takes into account the product family design, the process design and the supply chain design at the same time. The product family design consideration is the determination of an efficient modules set allowing to assemble products and to avoid function redundancy.

Data		f = Square function										
η	T	Two-phases Approach							integrate approach			gain perc.
		Tot CF	Tot Cv	Z1	Z2	Objective function	Size1	Comp. Time (sec.)	Objective function	Size2	Comp. Time (sec.)	
1	2	5,5	245,4	250,9	14,8	265,7	21	27,5	265,1	23	4,8	0,21%
	3	2,0	170,3	172,3	25,0	197,2	15	2,5	190,9	19	0,36	3,22%
	4	1,1	131,9	133,0	29,8	162,7	11	0,2	158,4	13	0,2	2,65%
	5	0,7	113,0	113,7	31,1	144,7	9	0,2	144,1	10	0,2	0,45%
	6	0,4	108,7	109,1	31,9	141,0	8	0,2	141,0	8	0,2	0,00%
20	2	104,5	250,5	355,1	16,3	371,4	20	597	370,2	21	273	0,32%
	3	39,6	170,3	209,9	22,6	232,5	15	5,6	230,0	15	7,4	1,06%
	4	22,0	131,9	153,9	27,7	181,6	11	0,25	180,6	11	1,75	0,51%
	5	13,2	113,0	126,2	31,1	157,3	9	0,2	157,3	9	0,2	0,00%
	6	8,8	108,7	117,5	31,9	149,4	8	0,2	149,4	8	0,17	0,00%
40	2	209,0	250,5	459,6	15,9	475,5	20	269	475,5	20	577	0,00%
	3	79,2	170,3	249,5	24,9	274,3	15	7,7	269,6	15	13,7	1,72%
	4	44,0	131,9	175,9	28,3	204,2	11	0,3	202,6	11	1,9	0,77%
	5	26,4	113,0	139,4	31,1	170,5	9	0,2	170,5	9	0,2	0,00%
	6	17,6	108,7	126,3	31,9	158,2	8	0,2	158,2	8	0,3	0,00%
60	2	313,5	250,5	564,1	15,9	580,0	20	501	580,0	20	760	0,00%
	3	118,8	170,3	289,1	24,3	313,3	15	10	309,2	15	14,8	1,32%
	4	66,0	131,9	197,9	27,2	225,1	11	0,3	224,6	11	1,5	0,20%
	5	39,6	113,0	152,6	31,1	183,7	9	0,2	183,7	9	0,22	0,00%
	6	26,4	108,7	135,1	31,9	167,0	8	0,2	167,0	8	0,23	0,00%
80	2	418,0	250,5	668,6	15,9	684,5	20	204	684,5	20	710	0,00%
	3	154,0	174,0	328,0	20,4	348,3	14	10,75	347,8	14	18,2	0,16%
	4	88,0	131,9	219,9	29,8	249,7	11	0,3	246,6	11	0,4	1,22%
	5	52,8	113,0	165,8	32,2	198,0	9	0,2	196,9	9	0,22	0,57%
	6	35,2	108,7	143,9	31,9	175,8	8	0,2	175,8	8	0,2	0,00%

Figure 5: Results for  $f(q_j) = (q_j)^2$

The process design consideration is delivery time constrained. Finally, the supply chain consideration is capacity constraints of distant location facilities.

The model objective functions consist in optimizing the costs occurring in the activity of the producer and the suppliers. The main result is that the module architecture depends specially on cost configurations between process and supply chain and depend also on the delivery time.

Further tests are conducted at this time in order to analyze the influence of the other problem parameters. For big instances, we intend to develop and test some heuristics because the complexity of the problem is too important.

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